

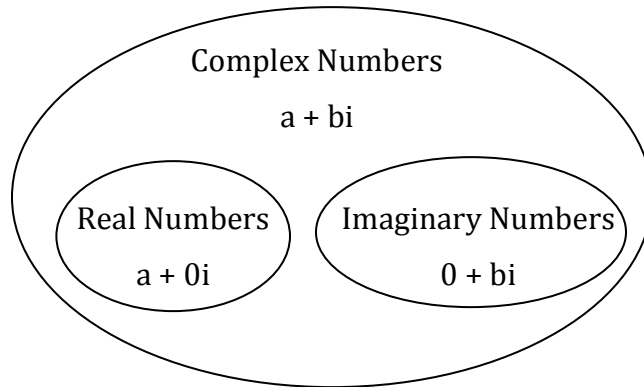
# Complex Numbers in Teacher Education: Connecting Mathematics and Pedagogy

Complex numbers seem to be disappearing from the secondary curriculum across the world. However, complex numbers provide a broad and useful domain of inquiry for teacher preparation. This set of complex numbers offers teachers a forum in which they can revisit secondary mathematics concepts, take those concepts to a deeper level in their classrooms with students than they might otherwise, and make explicit the pedagogic implications of teacher choices with regard to content, method, and sequencing of teaching. Additionally, examining complex numbers at the teacher education level provides an opportunity to teach mathematics in a way that models research-based classroom practice.

The mathematical notions linked to complex numbers include sets of numbers and field properties, history of mathematics, trigonometry, polar coordinates, vectors, radicals, operations on binomial expressions, polynomial functions and equations, geometric transformations with or without matrices. As well, a study of complex numbers allows for an examination of the ways in which different mathematical representations inform each other.

## Learning about Number Sets and Their Place in the History of Mathematics

One of the fundamental questions in mathematics was “how many.” The development of counting numbers answered this question. The idea of the absence of an item or a quantity led to the concept of zero. Then, the ideas of forward and backward, above sea level or below sea level, profit or loss, etc., led to the development of negative numbers. Similarly, solutions to varieties of equations or specific problems led to the development of additional number systems: rational, irrationals, and reals. For example, the need to find solutions to equations such as  $x^2 - 2 = 0$  led to the *reals*. Continuing, the need to find solutions to certain polynomial equations such as  $x^2 + 1 = 0$  led to the idea of  $x = \sqrt{-1}$ . To be able to talk about the solutions of such equations, a new number system had to be developed, which involved  $i$ , an imaginary number. Imaginary numbers combine with the real number system to give us the complex number system. (See Figure 1.)



**Figure 1**

Prospective teachers need a strong knowledge of number systems and their interrelationships. Such interrelationships include the operations on complex numbers and their properties, or lack thereof; for example the set of complex numbers, unlike the set of real numbers, is not ordered.

Various ways of representing complex numbers need to be firmly established. Diverse representations provide the vehicle for understanding calculations and also for connecting complex numbers conceptually to other areas of mathematics, for example, trigonometry, vectors and geometry.

### **Probing Secondary Mathematics: Complex Roots of Polynomial Equations**

As done historically, one area for extending secondary school mathematics with the study of complex numbers is the solution of polynomial equations. In the study of complex numbers, the conceptual difference between a complex plane (in which points represent single numbers that can be added, multiplied, etc.) and a Cartesian plane (in which points represent ordered pairs of numbers) can be highlighted. The visual appearances of the two representations are similar, and teacher awareness of the possible consequences of this similarity for student learning is part of pedagogical content knowledge.

### **Teacher Knowledge of Student Technical Difficulties**

Teacher pedagogical knowledge of complex numbers includes familiarity with the particular difficulties students exhibit in calculations with complex numbers. Some of them may not be unique to the topic itself. Two technical difficulties are the completion of squares and finding the argument when going from rectangular to polar coordinates. As well, students often mistakenly think that  $(x + y)^2 = x^2 + y^2$ , applying the distributive

property where it is not appropriate. Complex numbers provide another space in which to address binomial expansion.

### **Conclusion**

Complex numbers can be used to facilitate learning of mathematics for prospective teachers and in professional development settings. Connecting complex numbers to other areas of mathematics can be a vehicle for learning other mathematics. The topic could be a major part of a capstone course in the undergraduate preparation of teachers; for existing teachers, inservice in this arena could serve to broaden and enrich their teaching of mathematics.