

Institute for Advanced Study/Park City Mathematics Institute
International Seminar: Bridging Policy and Practice in the Context
of Reasoning and Proof
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**Conditions for Teachers to Engage in Problem Solving
and Reasoning in Their Classrooms**

Problem solving and reasoning are crucial in developing mathematics skills and competencies. Conditions affecting the teaching and learning of problem solving and reasoning include the following:

Assessments — Since problem solving and reasoning are central to mathematics learning, we recommend they be assessed in both internal and external examinations. Tests may be expanded to include problems that are non-routine, open-ended and/or from real life situations. While tests on formal proofs may be limited to the advanced courses, tests on pre-formal and content-related reasoning should be tested at all levels. Rubrics should be developed and used for grading problem solving, reasoning and proof to ensure reliability.

Class size — Problem solving and reasoning should be taught regardless of class size. In the case of large classes,

- Students could be divided into smaller groups with group leaders helping their peers. The teacher should ensure that leaders are well motivated and trained in how to manage mathematical content. Student leaders should also have some techniques for working with groups. The teacher should be prepared to monitor what happens within all the groups.
- Giving long-term assignments will allow students to explore and reflect on possible solutions. This may also help with time issues mentioned later.

Teacher training — Pedagogical training must include the strategies involved in problem solving and reasoning. The use of technology, techniques of assessment in the classroom, some theoretical background, and sharing and learning from each other are important. A mentoring program, in-service training, and participation in seminars and conferences should also be part of helping teachers learn to use problem solving in their classrooms.

Cultural norms — Different cultures and countries may have different approaches to problem solving, reasoning and proof. These approaches could affect how teachers and students study and learn mathematics. Examples include the following:

- Peer pressure may discourage certain groups of students (females or very bright students) from engaging in these mathematical activities.

- Less motivated teachers may discourage other teachers from the creativity and inventiveness needed to try different approaches and methods to engage in problem solving, reasoning and proof.
- Traditions of memorization, copying notes and exercises may work against the inclusion of more open-ended problem solving and reasoning.
- In-grained teacher or cultural beliefs that all students cannot engage in problem solving, reasoning and proof activities may be a detriment to student learning.

Despite the potential barriers, we strongly recommend that learning through problem solving, reasoning and proof be part of the curriculum for all students. The recommendation is made knowing that change does not occur rapidly, and new ideas and approaches may have to be introduced gradually. One source of advice regarding effective approaches to introducing new ideas is seen in the *Adapting/Adopting Best Practices*, a brief produced during the PCMI International Seminar meeting in 2005. Another possibility is visiting and learning from other countries.

Support for change — Changes can take place and take place rapidly if there is general support for the change. Hence it is essential for teachers to work to ensure that authorities in schools, parents and others concerned about education understand the need for problem solving, reasoning and proof in mathematics.

Syllabus — Problems solving must be an explicit part of the national curriculum or standards if it is to be valued and taught in the classroom. Problem solving must also be valued at the college and university levels, and creativity and problem solving must be incorporated in the entrance examinations to reflect that importance.

Time — Teachers should be aware that problem solving and reasoning are time consuming. Therefore, we recommend that some topics in the mathematics curriculum be reduced in order to give space for student exploration, and that efforts are made to design the development of mathematical concepts in ways that involve problem solving, reasoning and proof. This will affect teachers' planning and preparation for problem solving, reasoning and proof. A classroom teacher needs some flexibility in the allocation of time to teach within a classroom depending on student needs in learning problem solving, reasoning and proof. While teachers are accountable to the system in which they work, they have some control of the syllabus in their own classrooms.

Technology — Technology can enhance the teaching and learning of problem solving, reasoning and proof. A variety of non-routine problems may be approached with the use of technology. When possible, technology should be used to enable students to experience problem solving, reasoning and proof.

Language — Teaching problem solving, reasoning and proof demands effective communication among students and teachers. Contexts and brainstorming ideas for problem solutions along with the reasoning involved place increased demands on teachers to work to overcome any language difficulties. Teachers and researchers need to examine issues locally to see what needs to be done with language to enhance learning.

To communicate mathematical ideas to others, it is eventually necessary for students to use precise mathematical language.

Fear of change — A teacher with minimal knowledge of mathematics content and pedagogy may not have the confidence to incorporate problem solving and reasoning in the classroom in more than a peripheral way. Such a teacher may not be confident enough to accept different student answers, approaches and reasoning when non-routine problems are introduced. A fear of repercussions from authorities and parents for introducing new problems and content may impede implementation. Mentoring may help eliminate such fear.

Lack of resources — Teachers and students should have access to a variety of resources including textbooks, technology (hardware and software), materials for problems, and people with whom to discuss problems, including everyday life problems. Teachers may need to share existing resources. Teachers have to be innovative and improvise with existing resources. Teachers are encouraged to look for possibilities of sharing resources among schools in their locality and from external sources.