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Problem-Solving in the School Mathematics Curriculum

Why use problem solving?

Many reasons for including problem solving in the school mathematics curriculum have been discussed in many places (e.g., Cockcroft, 1982; NCTM, 2000). Essentially, problem solving is a natural process in the development and application of mathematical knowledge, which stimulates and develops critical thinking and, in particular, higher order thinking skills such as analysis, synthesis, evaluation and judgement.

Problem solving can be used to motivate students by providing them with interesting situations that represent a challenge and that often relate mathematics to real-life situations. It offers the opportunity to "use knowledge meaningfully" (Marzano and Pickering, 1997, pp. 1-6). Problem-solving can also be used effectively as a vehicle for introducing new mathematical knowledge to students by creating a need for that knowledge. Its use in this context can allow students to acquire a deeper conceptual understanding of the topic through their active participation in building their understanding.

The use of problem-solving can place both teachers and students in a position of uncertainty, often accompanied by a degree of anxiety, which if appropriately handled can stimulate effective learning and foster creative thinking.

Problem solving breaks the paradigm of thinking about mathematics as a "magic box of tricks" to be applied in a mechanical fashion. Students should be encouraged to reflect on the strategies and tools that could be used before embarking on the process; it is important to avoid the danger of replacing one set of tricks with another. Problem solving forces the student to become an active participant in the learning process and can make mathematics more accessible to students by allowing them to take part in the construction of their own knowledge.

What is problem solving? (in the context of mathematics education)

> The resolution of any task for which the student does not have an immediate method available and which may lead to the development of new mathematical content or processes through a non-linear approach.

This definition is clearly dependent on context. Depending on the experience and mathematical knowledge of individual students, the same problem may represent different challenges for each student.

The purpose of problem solving

- To develop a new piece of mathematics

  In this case the teacher retains a high degree of control over the process in order to ensure that students reach the desired end point. These problems may be simple in nature but must be very carefully selected in order to ensure that the fundamental mathematical idea emerges.

- To apply mathematics in a previously unknown way

  In this case the teacher can relinquish much more control to the student, giving them flexibility and ownership of the process.

  - Focus on process

    In some problems the end result in itself is not as important as the process required to obtain it. This does not mean that incorrect solutions are acceptable but rather that the purpose of the problem is primarily to encourage reflection on the strategies used.

  - Focus on result

    It is important for students to experience the application of mathematics to the real world, and in this case it is necessary to evaluate the solution in the real context to see whether it is viable.

- To foster the social, emotional, personal and intellectual development of students

The nature of problem solving

Problem solving implies some or all of the following: exploration, experimentation, collaboration, communication and perseverance, within a systematic framework (plan) that is subject to constant revision and modification. The role of previous knowledge and experience is important, and the use of problems that are similar in nature to previously seen work allows students to draw critically and selectively on their mathematical knowledge, thus establishing connections and relationships.

Characteristics inherent to the nature of problem solving, which contrast with passive learning, are the non-linearity and unpredictability of the process. These might cause some students and teachers to resist the introduction of problem solving in the mathematics classroom. Further contrasts are found in the variety of approaches that students can employ (Burton, 2002) and the multiplicity of methods of solution that are often available for a given problem.

The experiences of seminar participants suggest that the uncertainty implicit in problem solving is an obstacle to its widespread use. Vygotsky's theory of the zone of proximal development (Vygotsky, 1962) suggests that appropriate scaffolding is necessary for students to feel able to approach the problem but that in excess it can destroy the problem

by taking the challenge of looking for a suitable strategy away from the student. Since this zone depends on the individual student, there is clearly an enormous challenge for the teacher in providing the right amount of support for each student. The challenge of providing support for students in the learning process without overly scaffolding has emerged in recent studies as a barrier to real student learning (Stein et al, 2000; Stigler & Hiebert, 2004). Fragile teacher knowledge is also recognised as a major obstacle (compounding the uncertainty element in the teacher). This issue is discussed later in this report (see the brief titles *Conditions for Problem-solving, Reasoning and Proof*.

The process of problem solving

The process of solving mathematical problems involves:

- Understanding and defining the problem
- Exploring the problem
- Hypothesising
- Testing
- Formalising
- Reflecting/evaluating

It is important to note that these stages are not hierarchical in nature but rather form part of a complex cyclic process. In many instances of general practice the stages involving understanding the problem and reflection on the process tend to be weak, with a trivialising of these stages being common. Ways of strengthening and deepening the reflection process could be the use of problems with more than one solution and an emphasis on reflecting on the process, as well as on the result, with a view to generalising the learning that has taken place. Teacher scaffolding often cuts exploration short, and hypothesising was also considered to be a weakness in some countries.

Recommendations

- Look for ways to use problems, where possible and appropriate, to introduce new mathematics.
- Encourage the problem-solving approach as a way of approaching mathematics and not as a topic to be covered in isolation.
- Recognise and provide for well-designed and appropriate resources for the introduction of new subject knowledge through problem solving.

References

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