Objective: This activity allows students to develop and carry out a simulation, and analyze results in terms of mean and median.

Note: The cereal box problem is intentionally open-ended to create a task with a high level cognitive demand.

Student Background:
- know and understand how to calculate mean and median
- know how to use a random sample
- (Optional) know how to use required technology for the simulation, either a graphing calculator, or a computer with Fathom II software

Equipment needed:
- Cereal Box Worksheet (one for each student)
- Dice (for each group of students)
- (Optional) TI – 84, TI – 84+ calculator or other graphing calculator
- (Optional) Computer with Fathom II software

Lesson:
Provide each student with a copy of the Cereal Box Worksheet. Discuss the number of boxes of cereal students think would have to be purchased in order to collect the six different toys. Place students in small groups and have them design a simulation to analyze the problem. Allow time for them to complete at least one simulation using their design.

Possible solutions:
Students will possibly use many different simulations. Several are presented here including rolling a die, using a graphing calculator, and using Fathom II.

Solution 1: Rolling a die
Each distinct toy is assigned a number from 1 to 6. Students roll a die and keep a list of the number appearing on the face. How many rolls are required until all numbers six numbers have appeared? A completed example of a set of rolls to obtain all six numbers is seen in Table 1.

Table 1: Simulation of Cereal Boxes Bought to Obtain Six Toys

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>//</td>
<td>//</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>///</td>
</tr>
</tbody>
</table>

Based on this simulation, 10 boxes of cereal would have to be purchased in order to get all 6 toys. It could be possible to purchase as few as 6 boxes but that
is unlikely. It is useful for the each person in the class (or group of students) to conduct the experiment 10 times, and combine the data and determine the expected number of boxes required.

**Solution 2: Using a TI-84 or a TI 84+**

Students may use the random integer generator on a graphing calculator, write a program, or use the APPS Prob Sim to help decide on the expected number of boxes of cereal purchased to collect all 6 toys.

Below is an example of a program a student might submit. [This example program was written by an eleventh grader.]

```
0→A:0→B:0→C:0→D:0→E:0→F:0→I
While A=0 or B=0 or C=0 or D=0 or E=0 or F=0
randInt(1,6) →R
Disp R
If R=1:Then:1→A:End
If R=2:Then:1→B:End
If R=3:Then:1→C:End
If R=4:Then:1→D:End
If R=5:Then:1→E:End
If R=6:Then:1→F:End
I+1→I
End
Disp "IT TOOK: ",I," BOXES"
```

**Solution 3: Using Fathom II:**

A student might follow the following steps.

1. Open a new Fathom document.
2. Drag a new case table from the shelf and create an attribute called face.
   Change the name of *collection1* to *toys* as seen in Figure 1.

**Figure 1: Case Table**
3. Create a formula for the attribute face. Select Edit > Edit Formula. Type in `randomPick(1,2,3,4,5,6)` as seen in Figure 2.

**Figure 2: Case Table with Formula for randomPick**

```
<table>
<thead>
<tr>
<th>toys</th>
<th>face</th>
<th>&lt;new&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>randomPick (1, 2, 3, 4, 5, 6)</td>
<td></td>
</tr>
</tbody>
</table>
```

4. Choose Collection > New Cases. Ask for 1 case (the default) as in Figure 3.

**Figure 3: Case Collection with Default Case 1**

```
<table>
<thead>
<tr>
<th>toys</th>
<th>face</th>
<th>&lt;new&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>randomPick (1, 2, 3, 4, 5, 6)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
```

5. Highlight the collection `toys` as in Figure 4.

**Figure 4: Collection of toys**
6. Now choose Collection > Sample Cases as in Figure 5. Be sure the sample tab is showing; select “With replacement”, “Replace existing cases”, and “until condition”.

Then double click on the white rectangle to enter the formula
uniqueValues(face)=6 (Either type it in or select the uniqueValues function for the Functions list in Statistical/One Attribute). This instruction tells the program to sample until the values of 1, 2, 3, 4, 5, and 6 have all occurred. A new collection called Sample of toys will be created with this instruction.

**Figure 5: Sample of toys Collection**

![Inspect Sample of toys](image)

7. Expand the Sample of toys icon in Figure 6.

**Figure 6: Expanded Sample of toys Icon**

Each blue ball represents a “box” of cereal purchased in Figure 7.

**Figure 7: Blue Balls Representing Boxes Purchased**
8. To see the result of each purchase, include a case table for *sample of toys* as in Figure 8.

**Figure 8: Case Table for sample of toys**

<table>
<thead>
<tr>
<th>Sample of toys</th>
<th>face</th>
<th>&lt;new&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
Note: The second collection Sample of toys simulates the rolling of a die until all 6 numbers have been rolled at least once. The Measure called totalrolls keeps track of the number of rolls required for all 6 numbers to have appeared once.

10. With the Sample of toys selected, choose Collection > Collect Measures. Another collection called Measures from Sample of toys will be created in Figure 10.

Figure 10: Collection—Measures from Sample of toys

11. Double click the collection in Figure 10. Select the Measures tab. Create 3 new measures, meanrolls, minrolls, maxrolls. The formulae are mean(totalrolls), min(totalrolls) and max(totalrolls) respectively in Figure 11.

Figure 11: Inspecting Measures from Sample of toys
12. Select the Collect Measures tab from the Inspect Measures from Sample of toys Inspector of Figure 11. Be sure Replace existing cases is selected, Animation on is NOT selected and 500 measures is checked in Figure 12.

**Figure 12: Inspect Measures from Sample of toys with Collect Measures Chosen**

13. Click on Collect More Measures to simulate purchasing boxes of cereal so that 500 sets of 6 toys are collected.
14. Make sure the collection *Measures from Sample of toys* is selected in Figure 13. Drag down a new case table. Be sure there are 500 cases, or double click on the collection to have the inspector appear and select the **Cases tab** to show the number of rolls required for one case (usually the first). At the bottom of that screen will be 1/500 indicating the first case of 500 cases as in Figure 14.

**Figure 13: Measures of Sample of toys**

![Measures from Sample of toys](image1)

**Figure 14: Inspect Measures from Sample of toys**

![Inspect Measures from Sample of toys](image2)

15. Create a histogram with **totalrolls** on the horizontal axis as in Figure 15.

**Figure 15: Histogram Showing totalrolls**
16. Choose Graph > Plot Value, and type in `mean()` as in Figure 16.

**Figure 16: Histogram of Sample of toys with Mean**

17. Choose Graph > Plot Value, and type in `median()` as in Figure 17.

**Figure 17: Histogram of Sample of toys with Mean and Median**