Reflection on Improving the Lesson

Don’t Fence Me In: Area of a Parallelogram

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Trial Teaching of the Lesson

The following describes the discussions and revision process that produced the final lesson plan.

The first plan was conducted as a practice lesson for SSTP participants. The test lesson was very useful as we noticed difficulties that detracted from our lesson goal and caused us to make revisions to the lesson. The directions for the introductory problem were unclear. Many students were confused by the 5, 8, 5, 8 constraints. Some students overlapped strips and the lengths changed as a result, and some groups broke the fence into pieces. To avoid this, we decided to use spaghetti or straws instead of strips of paper. We agreed that we needed a visual representation of the rancher problem. The problem needed to be written down so students could refer back to and reread the problem. Receiving the information verbally once was not enough and led to questions that could have been avoided. Using the area of 38 in stating the problem confused some of the students so we decided to leave it out of the problem statement and use it later after students had found areas different from 40. We removed the term “environmentally conscious” because it led to some questionable ideas about cramming animals into confined spaces. Too much time was spent in groups, which did not leave enough time for pulling the class together for a discussion. We did not formalize the area of the parallelogram as a class.

When we handed out the worksheet with all the parallelograms that were supposed to have the same area, it turned out that one parallelogram did not have the same area as the others. It was an error in the worksheet. We discussed whether or not we should change it. Ultimately we agreed to correct the mistake because we wanted to drive the point home it. Ultimately we agreed to correct the mistake because we wanted to drive the point home that if the perpendicular measures are the same, the area is the same. Handing out graph paper and tools took time, so we decided to prepare a bag of materials containing straws of length 5 and 8, compasses, rulers and protractors and paper and graph paper for each group and to arrange the desks in groups before the students arrive. (See Lesson, Version II.)

Teaching the Lesson

Today Jo taught the lesson to the high school geometry class while the rest of the group members and facilitators observed. We began the debriefing by reviewing our goal -to emphasize that perpendicular segments are the measures that determine the area of quadrilateral and not necessarily the side measures.

After reflecting on the lesson, everyone agreed we needed to address the issue of time. The class started at 9:07, and most of the class time was spent working in groups. It was only at 9:50 that the students completed the task, and group discussion began. We were not able to get as far in class discussion as we would have liked, and we did not get to the parallelogram worksheet we were hoping to do. Towards the end of the lesson, the class was finally discussing and addressing the key concepts and making connections we were hoping to make as a group. If we had more time at this stage in the lesson, we all agreed the lesson would have definitely been more effective.

We discussed what happened during the group work to cause this time strain. The lesson seemed to overestimate the students' prior knowledge. We assumed they would know and remember the formula for the area of a parallelogram, but most of them did not. The original rancher problem now had much clearer constraints and visuals (in comparison to Friday's test lesson); however, students still spent much time veering from the topic. Their initial reaction was that the task was impossible. They spent a great deal of time creating other quadrilaterals and triangles and took a long time to form a parallelogram. Many of them ruled out the parallelogram right away because they assumed it had the same area as the rectangle. This was a mistake we saw in all the groups. Some students finally remembered the formula *bh* but then immediately said, "Oh, it's the same area as the rectangle." They simply multiplied 5x8. They clearly did not know what a base was and what a height was. Some groups drew in the height as 5, but then the side measure was clearly not 5 and so they were not following the constraints of the problem. We decided to spend more time asking questions like, "What do you mean by base? or "What is height?"

Posing the problem

We decided to hinge the sides of the fence together in the manipulatives so that the students would not waste time forming other quadrilaterals like kite or a trapezoid (using straws and pipe cleaners, for example). This would guide students to a parallelogram much earlier in the process. We also agreed we should provide a hands-on manipulative for each student. This would remove the issue of the students creating a parallelogram with a height of 5 and base of 8 but a different side length.

Since many groups had the same misconceptions, we decided to change the questions slightly to ask first what other shapes could be made using the hinged model. After the class agrees that the shape is a parallelogram we will ask students to find the area, which should eliminate much of the wasted time on the type of shape that is created.

Exploring the mathematics

The observers noted that group dynamics was a problem. In one group, one student immediately came up with the answer; however, the group did not discuss his reasoning, and it became very clear that not everyone in the group understood their answer when Jo called on a group member, and the student was not able to explain the answer. Furthermore, the student who did get the answer did no further work, and when Jo suggested finding another area, he said there was no other area. We discussed alternate questions to ask, such as "How many possible areas could you make?" "Is this the only parallelogram you can make with these side measures? Explain." One solution to the problem of group dynamics would be to have the students work in pairs, which would allow students to become more active participants. Another suggestion was to ask questions that would help facilitate discussion among the group members. One question that needed to be written in the original problem is "justify" or "explain how you found your area."

The directions to students when making their posters should include drawing their figures large enough so everyone can read the labels. Observers noted that some students felt they had to use all of the resources provided for them in their bag of materials ("They would not have given us this if they did not want us to use it."), and they wasted time trying to figure out how to use them. We decided to put the resources on a table at the back and just indicate they were there in case anyone wanted to use them. We also needed to supply a yardstick.

We suggested that the lesson could include a few leading questions for the rectangle problem:

- How did you find the area of the rectangle?
- Why does multiplying 5 by 8 give you the area?
- What is the relationship between the sides?
- What is base and what is height? (This question could really be asked several times, even during the parallelogram problem.)
- Have the students draw the rectangle on grid paper and use that as a tool to explain the method of solving for the area.

One group of students (Group B below) used the Pythagorean Theorem to calculate the height to be 4.6 and then rounded to 5. They concluded that the area was then 40 again. Jo questioned them, and they were able to resolve this issue. The comment was made that perhaps Jo could have discussed this issue with the class (time permitting).

Sharing strategies

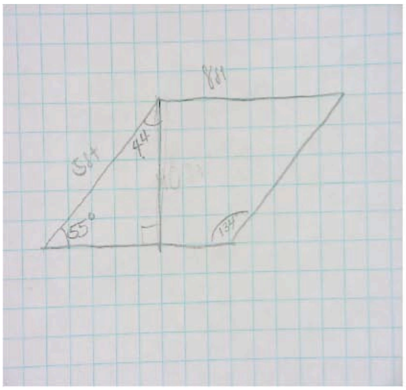
Some students seemed to be dependent on being told or simply "knowing" the answer or formula without understanding what it meant. Many students came up with the parallelogram but could not remember the formula for the area. Some students used their text, notes and resources; Group A divided the parallelogram into two triangles and used the area of a triangle formula from trigonometry, $A = \frac{1}{2}ab(\sin C)$, to find the area of the parallelogram. While the use of this formula was very interesting and a great method of solving the problem, we did not have time to question whether or not these students really understood where the formula came from and why it works. This solution also did not directly highlight the use of perpendiculars in finding area, so we would need to consider how to keep the discussion from going away from our goal.

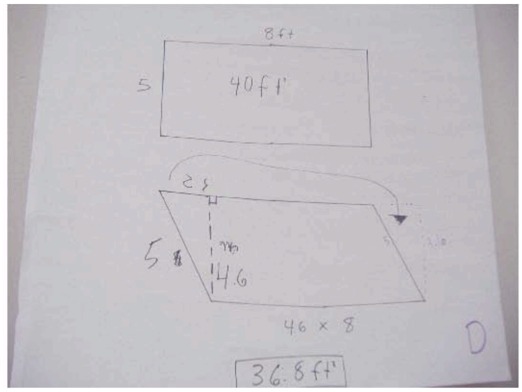
The student posters showed only the final result for each group, and their thought processes and work were not visible to share. Many groups started with incorrect reasoning and assumptions that would have been valuable to discuss with the class. Several solutions were suggested: to use a "placemat" where each student's individual work is in one corner of a large sheet of paper, with the final result in the center; another was to have the teacher note the different strategies and ask general questions about them: "Will it work to find the area by taking the product of the sides? Why or why not?" "One strategy I thought about was to measure the angles and use the trigonometric ratios. Would this work?")

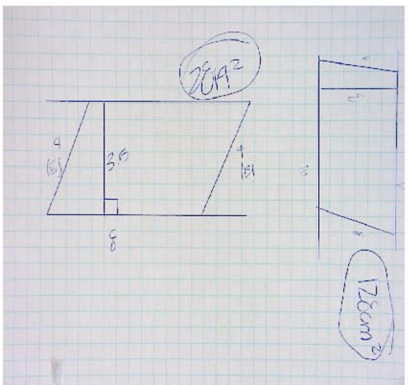
One observer noted that one group had elements of a great discussion but did not pursue the ideas. They noted that the maximum area would be found in the rectangle and anything else would create a parallelogram with a height less than 5, and therefore a smaller area. This discussion should have been highlighted and developed further as we had intended to do later in the lesson, and we discussed how to facilitate or promote this discussion in all groups. One related observation was that the students did very little to justify their work.

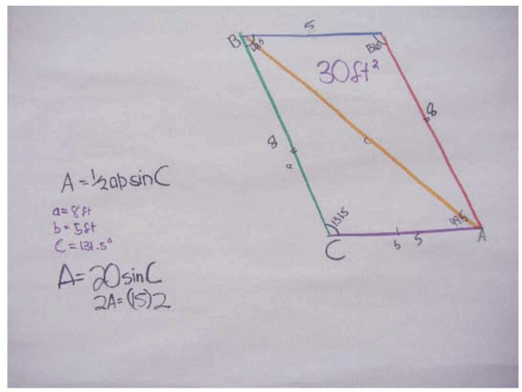
Another note on the student work was that when one group posted their solution, it came up "side-ways" because of the nature of the adhesive on the paper. We adjusted the orientation of the paper so that the base would be on the bottom. However, perhaps we could have left the paper oriented like that so we could have a discussion of base and height. We could use this opportunity to emphasize that the base does not have to be on the bottom. A suggestion was made that we could first turn the paper like the students originally wanted but then switch it around during the discussion in order to bring up this point. Observers also noted that two different orientations were shown for the rectangle, horizontal and vertical (either base of 8 or 5). However, what if the rectangle were displayed diagonally? Three of the groups used a base of 8 in their parallelogram and only one group used 5; the lesson should instruct the teacher to look for these differences and raise questions about how the figures and area are alike and different and why.

Student A Group B









Student C Group A

We collected and studied the student work and found we could discover more about student thinking. Student A measured the angles but did not realize that the degree sum of the angles in the triangle should have been 180 or that the adjacent angles in a parallelogram were supplementary. Some of the students (Student C, for example) did not use graph paper effectively as a tool; they drew a parallelogram but ignored the grid lines in doing so. We decided that having students draw the rectangle on graph paper and pointing out the grid lines would help alleviate this problem.

After much discussion we asked ourselves whether we accomplished our goal. We agreed that we got much closer with the class than we did during the test teaching but that the lesson still needed modification and re-teaching. We also agreed that we had learned to think more deeply about teaching and what we do as teachers that enable students to learn.