Hyperbolic Plane Discussion
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David Wright

1. Is there a natural unit to measure the length of an interval in the line?

2. What is the natural unit to measure the length of an arc on the circle? What is the unit of measure?

3. Is there a natural unit to measure the length of an interval in the plane?

4. Is there a natural unit to measure the length of a segment on a sphere?

5. What is the pseudo sphere? What is the hyperbolic plane? Is there a natural unit to measure lengths in the hyperbolic plane?

6. Laws of cosines and sines in the plane for $\triangle ABC$ with angles α, β, γ and sides a, b, c.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

7. Laws of cosines and sines in the sphere for $\triangle ABC$ with angles α, β, γ and sides a, b, c.

$$(i) \quad \cos\alpha = \frac{\cos a - \cos b\cos c}{\sin b\sin c}$$

$$(i') \quad \cos a = \cos b\cos c + \cos\alpha\sin b\sin c$$

$$(ii) \quad \cos a = \frac{\cos\alpha + cos\beta\cos\gamma}{\sin\beta\sin\gamma}$$

$$(ii') \quad \cos\alpha = \cos a\sin\beta\sin\gamma - \cos\beta\cos\gamma$$

$$(iii) \quad \frac{\sin a}{\sin\alpha} = \frac{\sin b}{\sin\beta} = \frac{\sin c}{\sin\gamma}$$

8. Laws of cosines and sines in the hyperbolic plane for $\triangle ABC$ with angles α, β, γ and sides a, b, c.

$$(i) \quad \cos\alpha = \frac{\cosh b \cosh c - \cosh a}{\sinh b \sinh c}$$

$$(i') \quad \cosh a = \cosh b \cosh c - \cos\alpha \sinh b \sinh c$$

$$(ii) \quad \cosh a = \frac{\cos\alpha + cos\beta \cos\gamma}{\sin\beta \sin\gamma}$$

$$(ii') \quad \cos\alpha = \cosh a \sin\beta \sin\gamma - \cos\beta \cos\gamma$$

$$(iii) \quad \frac{\sinh a}{\sin\alpha} = \frac{\sinh b}{\sin\beta} = \frac{\sinh c}{\sin\gamma}$$

9. Area formulas for triangles

   Plane: $\frac{1}{2}$base $\times$ height

   Sphere: Angle Excess $\alpha + \beta + \gamma - \pi$

   Hyperbolic Plane: Angle Deficit $\pi - (\alpha + \beta + \gamma)$

10. Tessellate the hyperbolic plane (radius of curvature $r = 10\ cm$) with pentagons with five right angles. Start at a vertex and place four pentagons about the vertex this is layer 1 with a perimeter of 12 edges and 8 corners. Fill in the table with headings:

    Layers
    Angles in Perimeter
    Edges in Perimeter
    Pentagons in Layer
    Total Number of Pentagons
    Perimeter (km)
    Area (km^2)