11 *What Affine Day...*

PROBLEM

The government of a small country (Texas) has purchased 10,000 vials of vaccine, a 14-day supply. The living culture for the vaccine grows at a rate that produces 5% more vials of vaccine each day. In order to be efficient, it has been determined that the same amount of vaccine should be distributed each day.

(a) Write an iteration rule that describes this situation.

(b) How much vaccine should be distributed each day to exhaust the supply of vaccine in 14 days, when a new shipment will arrive?

(c) Complete this table with different options for Texas's buying power:

Purchase	Daily Distribution
9000	
10000	
11000	
12000	

Notice anything?

After all, Texas can decide at any time to divide up into as many as five states. It says so! Don't mess with Texas' vaccine.

Useful Stuff.

1. Oscar buys a house. He takes out a 15-year mortgage on the $200,000 cost of the house. The interest rate is 6.5%.

(a) If Oscar had an interest-only loan, how much would he have to pay each year?

(b) Instead, Oscar pays \$16,000 per year. Will he pay off the mortgage in 15 years? How much is left?
(c) Oscar jacks it up to \$18,000 per year. Now what happens? \$20,000?
(d) Determine the correct annual payment Oscar should make, based on these other results.

2. Judy considers the iteration rule $x \mapsto 2x$.
(a) What's the fixed point?
(b) You start 16 away from the fixed point. After taking the iteration once, how far are you away from the fixed point?
(c) After taking the iteration twice, how far are you away from the fixed point?

Sixteen away? Couldn't that be two numbers? Oh, wouldn't it be nice if they both came out the same...

3. Josue considers the iteration rule $x \mapsto 2x + 3$.
(a) What's the fixed point?
(b) You start 16 away from the fixed point. After taking the iteration once, how far are you away from the fixed point?
(c) After taking the iteration twice, how far are you away from the fixed point?

4. Susan drinks TaB while considering the iteration rule $x \mapsto 0.75x + 46.8$.
(a) What's the fixed point?
(b) You start 16 away from the fixed point. After taking the iteration once, how far are you away from the fixed point?
(c) After taking the iteration twice, how far are you away from the fixed point?

TaB. Wasn't it delicious? And no calories too! If only there were caffeine-free TaB.

5. Tandy thinks about her mortgage while considering the iteration rule $x \mapsto 1.06x - 7000$.
(a) What's the fixed point?
(b) You start 16 away from the fixed point. After taking the iteration once, how far are you away from the fixed point? What is going on here?
(c) After taking the iteration twice, how far are you away from the fixed point?

6. So, what is going on here? Describe the behavior on any linear iteration rule $x \mapsto Ax + B$. Specifically, when will $x \mapsto Ax + B$ have an *attracting* fixed point?

Bad affine joke here, and a possible def.

7. Amy buys a \$100,000 house, and pays off the mortgage at

6% APR. She pays $9,000 per year.

(a) What's the fixed point?

(b) How far is Amy starting from the fixed point?

(c) After taking the iteration once, how far is Amy from the fixed point?

(d) After taking the iteration 15 times, how far is Amy from the fixed point?

(e) Find a *closed-form* rule for the amount of money Amy owes after n years, based on the "how far away from the fixed point" idea.

8. Ben Affleck won an Oscar for what movie?

Useful Stuff from a Different Angle.

Multiple solution methods? What is this, the 11:00 hour already?

9. Allen reconsiders the iteration rule $x \mapsto 2x$, and starts with $x_0 = 7$.

(a) Write the next four terms, but don't simplify anything. No exponents neither. Lots of parentheses, this.

The first one is $2(7)$. We'll leave the rest to you.

(b) Alright, fine, now you can combine stuff. What do you get?

(c) Write a rule for x_{10}.

10. Claudia reconsiders the iteration rule $x \mapsto 2x + 3$, and starts with $x_0 = 7$.

(a) Write the next four terms, but don't simplify anything. No exponents neither. Yeck?

The first one is $2(7) + 3$. We'll leave the rest to you.

(b) Alright, fine, now you can combine stuff. What do you get? Uh oh, what you gonna do about all them 3s!

(c) Write a rule for x_{10}.

11. Demian reconsiders the mortgage rule $B \mapsto 1.06B - 9000$, starting with initial balance $B_0 = 100000$.

Oh NO, not the letter B! What do we do if it's not x? Close the schools!

(a) Write the next four terms, but don't simplify anything. No exponents neither. Messy, but lovely.

The first one is $1.06(100000) - 9000$.

(b) Alright, fine, now you can combine stuff. What do you get? What do you do about all them 9 grands?

(c) Write a rule for B_n.

12. What was Aki's secret Iron Chef ingredient?

Neat Stuff.

13. Show that the mortgage rules from problem 7 and 11 always give the same result.

14. Describe what the matrix

$$\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

does to any point in the plane.

15. Truly ticked at the other matrix's superiority, the matrix

$$\begin{bmatrix} 0 & 1 \\ -21 & 10 \end{bmatrix}$$

says "Yeah, big hotshot, $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$. You're not the only one that can scale points, you know."
Find some points, other than $(0, 0)$, that scale when multiplied by this matrix. In other words, solve this system:

$$\begin{bmatrix} 0 & 1 \\ -21 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix}$$

There is more than one "family" of points. What does that mean? Heck if we know, we just write these sidenotes to distract you.

Relatively Useless But Still Pretty Neat Stuff.

16. Consider the iteration rule

$$x \mapsto \frac{2x^2}{x^3 + 1}$$

(a) Find all the fixed points.
(b) Classify each fixed point as attracting, repelling, or neither.

17. Steve stares at a Rubik's cube for a real long time without realizing it's been completed already. He starts looking at the red side, then flips to another (bordering) side by turning the cube one of four directions at random. *In the long run*, what's the probability that Steve stops and finds himself staring at the green side (which is opposite the red)?

A *really* long time, if you ask us.