6 *Enter the Matrix...*

PROBLEM

Start with the recurrence

$$t_n = 5t_{n-1} + 14t_{n-2}$$

If $t_0 = 5$ and $t_1 = 8$, find a closed-form rule for t_n or, if you prefer, find Bowen's laptop. Oops, it's found! Guess you're stuck doing the box problem, then.

If Train X is headed from Seattle to Cinti at 70mph and Train Y is headed from McAllen to Cinti at 80mph, what is the probability that you get this problem right?

Useful Stuff.

1. Where we're going, we won't need roads (but you will need to look at the map on the board).
   (a) According to the map, how many ways are there to get from Las Vegas to Salt Lake City in two steps?
   (b) According to the map, how many ways are there to get from Park City to Las Vegas in exactly three steps?
   (c) According to the map, how many ways are there to get from Las Vegas to Park City in exactly three steps?

2. Build a six-by-six table of numbers that you can use to convey all the information from the map. Be ready to describe what you did in detail.

3. Use the table of numbers in problem 2, and not the map, to build a six-by-six table of numbers that tells you how many ways you can get from city A to city B in exactly two steps. Again, be ready to describe what you did in detail.

Can you get from Vegas to Salt Lake City faster if you Excelerate?

4. Pick a number and perform this operation repeatedly:

$$x \mapsto \frac{14}{x} + 5$$

(a) What happens in the long run?
(b) Find all the numbers where the output equals the input.

5. Pick a number and perform this operation repeatedly:

$$x \mapsto 2x - 5$$

(a) What happens in the long run? Does it depend on the number you start with?
(b) Find all the numbers where the output equals the input.

6. Pick a number and perform this operation repeatedly:

$$x \mapsto 1.015x - 420$$

(a) What happens in the long run? Does it depend on the number you start with?
(b) Find all the numbers where the output equals the input.

Pick a question and perform the copy-paste operation repeatedly…

7. Pick a number and perform this operation repeatedly:

$$x \mapsto .75x + 46.8$$

(a) What happens in the long jog? Does it depend on the number you start with?
(b) Find all the numbers where the output equals the input.

TaB® is all we have to say for now. I hope we've piqued your interest. Remember, drink TaB® to keep tabs on your diet! Note: this sidenote kept us from writing good math problems for like 15 minutes.

Neat Stuff.

8. You invest \$500 at 4% APR (Annual Percentage Rate).
(a) How much money will you have at the end of five years?
(b) To the nearest year, how many years will it take to double your money?
(c) Write a rule in the form $x \mapsto \ldots$ to describe the relationship between how much money you have between one year and the next.

9. You invest $500 at 4% APR in an annuity. Which means you invest a new $500 at the start of each year, on top of what's already there.
   (a) How much money will you have at the end of five years?
   (b) To the nearest year, how many years will it take for you to have $10,000?
   (c) Write a rule in the form $x \mapsto \ldots$ to describe the relationship between how much money you have between one year and the next.

10. Recall this iteration on points:

$$(x, y) \mapsto (y, x + y)$$

   (a) Pick a starting point. What happens after 6 iterations?
   (b) If you start at (a,b), where are you after 6 iterations (in terms of a and b)?
   (c) Bev chose the starting point (5, -3). What happens after 6 Beverations? What happens after 12 Beverations?
   (d) Can you find another starting point that (mis)behaves similarly to Bev's?
   (e) ***Tougher.*** Can you find another starting point that never corrects its (mis)behavior?

TaB® is also a beveration.

(0,0) doesn't count, neither does your average one year old. Ah, but does the *median* one year old? Represent this in a stem and leaf plot.

11. Find a recursive rule that fits the sequence 1, 3, 4, 2, -4, -12, -16, -8, 16, …

12. What happens when you try to find a closed-form rule for the sequence in problem 11? Is there anything that can be done about this horror show?

13. Say, here's an interesting recursive rule:

$$t_n = 10t_{n-1} - 25t_{n-2}$$

   Find a closed-form rule when $t_0 = 2$ and $t_1 = 25$. Hm, this ain't as easy as it was s'posed to be.

Irrelevant Stuff.

14. Ask Bowen for the vertical strips. You'll be glad you did.

15. Or, ask Bowen for the world's most fun Sudoku puzzle. Maybe you won't be as glad? Wondering what Sudoku is, aren't ya?

16. Or, ask Ben about how he likes to stack wood. Don't worry, it's nothing criminal. It might even be relevant (Bowen says it isn't).

Okay, it's more relevant than the other two. Maybe even better than a problem about rabbits.

Tough Stuff.

17. Use a picture of the complex plane to explain what on earth went on in problem 11.

18. You have a rental car with unlimited miles, so you start taking random roads according to the map of the cities (well, Eureka is putting "city" loosely to say the least) we presented. At each city you pick a road at random that leaves the city. *In the long run*, find the probability that you find yourself in Eureka.

19. Find a closed-form rule for the recursive rule

$$t_n = 9t_{n-1} - 27t_{n-2} + 27t_{n-3}$$

with the sequence going 5, 30, 117, 378… Heh, heh, heh.

20. Make your own Tough Stuff problem and submit it to Bowen and Ben. We'll take the best ones and put them on future problem sets. Or maybe we won't, and we're just making you try The Open-Ended Approach.