How to Use a Calculator

This is the process to show a Web diagram for iteration on the TI-83 or 84:

1. Select MODE, then select **Seq** (sequence) instead of **Func** (function). This is done because the calculator considers iteration to be a recursive sequence, like this:

   $$u_7 = 0.75 \cdot u_6 + 1.5$$

   where the u_n are the iterates. Each iterate comes from the previous one.
   There is one significant difference: the calculator uses function notation for this. So, instead of u_n, it will use $u(n)$ throughout.
2. Now select Y=, and the menu has changed. It asks for $u(n) =$ instead of $y =$, and is asking for the recursive rule. So, one potential rule is

   $$u(n) = 0.75u(n-1) + 1.5$$

   Entering this rule will perform the iteration function $u \mapsto 0.75u + 1.5$.
3. Enter the initial value, listed as $u(n\text{Min})$:

   $$u(n\text{Min}) = \{.1\}$$

   The calculator will add curly braces around any number you type here.
4. On the FORMAT menu (2nd-ZOOM), select **Web** to display a Web diagram.
5. Other settings pertaining to the graph can be changed by selecting WINDOW. For Web diagrams, xMin, xMax, yMin, and yMax are the limits on the Cartesian coordinates displayed. For this example, try setting both x and y limits between -10 and 10 to comfortably display the example given here.
6. Select GRAPH, and two graphs should appear: the iteration function and the line $y = x$.
7. Select TRACE, then push the right arrow key repeatedly. This will draw the Web diagram step-by-step.

The u button is 2nd-7 on the TI-84, and n is the X/T key.

9 It, or Eight?

PROBLEM

It's *graph paper time*. Take a piece of graph paper, and draw a very large, accurate graph of $y = x^2$ that goes from $x = 0$ to 2, and $y = 0$ to 4. (Take points every 0.2 units if you need a guideline.)

Now, on the same coordinate grid, graph the line $y = x$.

Starting from the point $(0.8, 0)$, move vertically to the graph of $y = x^2$. What are this point's coordinates?

Then, move horizontally to the graph of $y = x$. Cawdinates?

Then, move vertically to the graph of $y = x^2$. Cawds?

Continue moving vertically and horizontally. What happens?

Graph paper time means half-price hot chocolate in the hall, and a free raffle entry for one Zome.

Useful Stuff.

1. Tobey repeats what's in the box, starting from the point $(1.1, 0)$. What happens?
2. Find all the fixed points for the iteration $x \mapsto x^2$, and determine whether each is attracting, repelling, or neither.
3. Watch our demonstration. Don't do problems 4 and 5 until you've seen it.
4. Stephanie is getting hooked on TaB. Use a calculator to make a Web diagram that illustrates what happens when Stephanie starts drinking TaB every two hours. (She starts caffeine-free.)

It was either TaB or phonics, and Stephanie chose TaB.

D. DNCOQ.

5. Use a Web diagram to decide whether or not each of these iteration rules has an attracting fixed point.
(a) $x \mapsto -.9x + 9.5$
(b) $x \mapsto 1.3x - 2$
(c) $x \mapsto x^2 - 1$

Hey, we don't care if you already know the answer here. Build a Web diagram.

6. Use graph paper to determine the effects of each of these matrices on our happy little L:
(a) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
(b) $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
(d) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
(e) $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

7. For each new L in problem 6, find its area. The area of the original L is 13 square Cals.

Although the L has 13 square Cals, TaB has none. Never had it, never will.

8. Find a matrix that does each of these things to our happy little L:
(a) Reflect across the line $y = x$.
(b) Reflect across the line $y = -x$.
(c) Rotate 90 degrees counterclockwise.
(d) Make an L with area exactly 100 square Cals.

Neat Stuff.

9. Consider *all* the iteration rules in the form

$$x \mapsto Ax + B$$

(a) Will there always be a fixed point?
(b) Find the fixed point(s) in terms of A and B.
(c) Can there be more than one fixed point?
(d) Based on what you've seen (with Web diagrams, TaB, and annuities), determine when rules in this form will have *attracting* fixed points.

10. Consider the iteration rule

$$x \mapsto \frac{6}{x} + 1$$

(a) Use a Web diagram to illustrate that $x = -2$ and $x = 3$ are both fixed points.
(b) Use the ZOOM tools to zoom in around the fixed point $x = 3$. And I mean ZOOM.
(c) Is the fixed point $x = 3$ attracting? Use the Web diagram to tell.
(d) Repeat for the fixed point $x = -2$. ZOOM, zoom, zoom in then decide whether it's attracting from what the Web diagram shows.

Perhaps this fixed point is broken? Oh, wait, now it's fixed.

11. Consider the iteration rule

$$x \mapsto \frac{x + \frac{5}{x}}{2}$$

(a) Use a Web diagram to illustrate that $x = -\sqrt{5}$ and $x = \sqrt{5}$ are both fixed points.
(b) Use the ZOOM tools to zoom in around the fixed point $x = \sqrt{5}$. And I mean ZOOM.
(c) Is the fixed point $x = \sqrt{5}$ attracting? Use the Web diagram to tell.
(d) Repeat for the fixed point $x = -\sqrt{5}$. Send it to ZOOM, then decide whether it's attracting from what the Web diagram shows.

ZOOM, Box 350, Boston, MA. . . 0 2 1 3 4. . .

12. Which of these iteration rules have an *attracting* fixed point at $x = 9$?

$x \mapsto 2x - 9$	$x \mapsto -2x + 27$	$x \mapsto \frac{18}{x} + 7$
$x \mapsto 9$	$x \mapsto \frac{9}{x} + 8$	$x \mapsto -x + 18$
$x \mapsto \frac{x^2}{9}$	$x \mapsto x^2 - 81 + x$	$x \mapsto \frac{2}{3}x + 3$
$x \mapsto \frac{4}{3}x - 3$	$x \mapsto x^2 - 72$	$x \mapsto x$

13. What's up with this iteration rule?

$$x \mapsto \frac{x + \frac{17}{x}}{2}$$

Does it have any fixed points? Attracting fixed points? Why would this rule have been useful before the calculator?

14. Who would ever consider the iteration rule

Alan would. Why, wouldn't you?

$$x \mapsto \frac{x^2 - 15}{2x + 8}$$

(a) What are the fixed point(s) for this iteration rule?
(b) Which of the fixed points is attracting? Which is repelling? What?

15. What's the monthly payment on a car if it costs \$10,000 and you're paying it off at 6% APR, monthly (that means 0.5% interest monthly), for 48 months? Take a guess, then make a better one.

Tough Stuff.

16. Use Newton's Method to find an iteration rule with three attracting fixed points (you can pick where).

17. Find the two fixed points for this iteration rule:

$$x \mapsto 2x + \frac{1}{x}$$

Are they attracting or repelling?

18. Consider *all* the iteration rules in the form

$$x \mapsto Ax^2 + Bx + C$$

(a) Will there always be a fixed point?
(b) Find the fixed point(s) in terms of A, B, and C.
(c) Can there be more than one fixed point? Can there be more than one *attracting* fixed point?
(d) Determine when rules in this form will have *attracting* fixed points.

21. (continued) What's the common ratio for this iteration rule?

That's not a typo, the first output of the iteration rule will be $2z - 1!$ The factorial is not a typo either.

$$x \mapsto 2z - \frac{1!}{x}$$

Take the starting point $x = 1$ to begin with.