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How many triangles (any size) are in this triangle?

Two levels for this problem:
Level 1: find the number of the triangles in the above diagram
Level 2: generalize for an n-dimension triangle. (Either a formula or describe a
method to get it)

Your write-up needs to include exactly how you solved the problem. This needs to be complete
enough that a person reading it could recreate your work. Please include dead-ends or other
things that didn’t work, even if it seemed like a waste of time. This is part of your learning
process. If you received help, please write who helped you and how they were helpful. Include
all your drawings, charts, tables and anything else that got you to your solution.

If you were unable to solve the problem or only got a partial solution, you can still do the write
up. Include where you got stuck or why you think you may not be correct.
Determine if the problem is appropriate

a. Is the problem rich in mathematical content and valuable mathematically?
   ➔ Topics: finding patterns; analyzing patterns; determining a general rule for size
   n triangle (or a description to find it); quadratic models; multiple representations: tables, drawings, charts, graphs.

b. Is the mathematical level of the problem appropriate for the students?
   ➔ Approachable at multiple cognitive levels; appropriate for different course
   levels (use a specific size for a lower level course; generalize in an upper level
   course)

c. Does the problem include some mathematical features that lead to further
   mathematical development?
   ➔ Finding a model for the general triangle, mathematical writing to explain how
   the problem was solved

2. Anticipating students’ responses…
   ➔ First response: “This is impossible!”
   ➔ Encourage students to break the problem into a smaller problem.
   ➔ Students might miss the “upside down” triangles
   ➔ Encourage students to organize their work
   ➔ Encourage systematic thought, rather than haphazard guesses
   ➔ Possible hint: look at upside-down triangles separately
   ➔ Using regression to estimate a solution and help find patterns within the
   problem.

3. Making the purpose of using the problem clear
   ➔ Excellent intellectual challenge
   ➔ Finding and recognizing patterns (Oh no! Not again!)
   ➔ Communicate mathematically
   ➔ Go from specific cases to a generalization (upper level)
   ➔ Practice using a systematic approach to solve a complex problem
The above sequence of numbers (1, 3, 6, 10, …) is a quadratic function.