2007.10 This Tastes Game-y

Game of the Day: "M&M Survivor"

Gather up 40 M&Ms together in a cup and toss them on a plate. Some will be M-side up, count these and return them to the cup. Eat the others.

i can has m&m?

If using other things such as pennies, ignore this last step.

Repeat a total of six times, and see how many survive. Survivors ready . . . go!

1. Complete this table with the number of survivors from your experiment:

We decided it would not be appropriate to play this with people, what with the eating and all. Plus if you weren't wearing a shirt with an M on it, you're in real trouble.

# Tosses	Survivors
0	40
1	
2	
3	
4	
5	
6	

2. What type of fitting curve might you expect for this data? If time permits, use technology to find and display the fitting curve.

Would Gloria Gaynor enjoy this game?

Important Stuff.

3. (a) Pick two positive integers at random. What, approximately, is the probability that the two integers do not share a common factor greater than 1?

Using positive integers feels natural.

(b) We looked at this equation on Friday:

$$p + \frac{1}{4}p + \frac{1}{9}p + \frac{1}{16}p + \cdots = 1$$

What's this p stand for?

This has no affiliation to the Sue Grafton book "P is for Peril." Yes, we know how to use the internets!

4. Evaluate this on the nSpire:

$$\frac{1}{\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}}$$

Say what? There's an infinity button?? Use **ctrl** and the weird book thing on the right.

Holy moly.

This page has set the record for Most Wacky Sidenotes.

5. Calculator skill time. Your calculator should be loaded with a document called "farey50", but if it isn't, here's what you do.

I can only imagine the graphing calculator games of this new generation. It will put "Drug Wars" to shame.

- Go to the top-level menu by continually hitting the **ctrl** button and the up arrow. This should take you to a list of documents.
- Find another calculator with "farey50" on it. This document is located in the "Examples" directory.
- Get a link cable. Attach the side marked A to the calculator you are transferring *from*, and the side marked B... well, yeah.
- On the calculator you are transferring *from*, highlight the "farey50" document.
- Select the TOOLS menu by hitting the **ctrl** button and hitting the HOME button in the very top right.
- Select option 1, "File", then option 5, "Send".
- Watch the magic as you now have "farey50" in the Examples directory on your own calculator.

Don't let your godmother catch you doing this.

Okay, everybody got it? Now open the "farey50" document within the Examples directory by clicking on it. The first page is a spreadsheet with the data and the coefficients for a best-fit quadratic. The second page is a scatter plot of the data with the best-fit quadratic overlaid. Wow, that's close!

Hit **ctrl** and up to get to the menu, then open the Examples directory if necessary. If it asks you to save the current document, make like Nancy Reagan.

In any case, decide how you would estimate the number of elements in F_{100}, the Farey sequence of order 100, and for F_{200}. Once your group comes up with answers, we'll let you know how close they are.

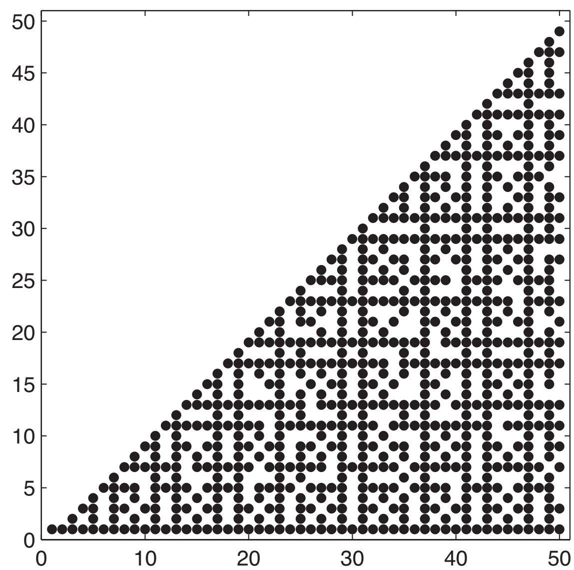
6. The best-fit quadratic for the first 50 Farey data points seems to be

$$f(x) = .3056x^2 + .2388x + 1.608$$

(a) As x gets larger, what terms become more important or less important in this quadratic?

(b) At the top of the next page is a plot of the points in F_{50}: if $\frac{y}{x}$ is in the Farey sequence, then (x, y) is plotted. Give an estimate for the probability that if I pick one of the 2500 points in the square $1 \leq x, y \leq 50$, it is in the Farey sequence.

Maybe a picture of F_{15} would be more helpful?



Neat Stuff.

7. Here's an interesting sequence of, uh, sequences.

Index	Sequence
1	1, 1
2	1, 2, 1
3	1, 3, 2, 3, 1
4	1, 4, 3, 2, 3, 4, 1
5	1, 5, 4, 3, 5, 2, 5, 3, 4, 5, 1

Each sequence *inserts* the index number n in all places where two consecutive terms in the sequence above add to n. The 5s are placed between 1 and 4, then 3 and 2, then 2 and 3, then 4 and 1. Anywhere it adds to 5.

(a) Build the next three sequences. In the first, insert 6s anywhere you see consecutive terms adding to 6.

(b) How many elements are in each sequence? Wowzers.

The heck, there was supposed to be a 17.

8. Look back at the lists of Farey sequences from earlier in the course.

For some students, this is their most favoritest fraction problem evah!

(a) What is the first fraction placed between $\frac{2}{5}$ and $\frac{1}{2}$?

(b) What is the first fraction placed between $\frac{2}{7}$ and $\frac{1}{3}$?

(c) What is the first fraction placed between $\frac{2}{5}$ and $\frac{3}{7}$?

(d) What is the first fraction placed between $\frac{3}{5}$ and $\frac{5}{8}$?

(e) What is the first fraction placed between $\frac{a}{c}$ and $\frac{b}{d}$?

9. Suppose you were standing at the origin $(0, 0)$ and there were 2500 points of light, one at every point (x, y) with integer coordinates 50 or less. You pan across from the east to the north, and in between you see a whole lot of points. You can't see any point blocked by another one: for example, you can't see $(10, 6)$ because $(5, 3)$ is in the way.

Sheesh, a Reagan and Bush reference on the same problem set? What have we come to? Still it's better than the "Over and Ova-Reagan" reference from 2005.

(a) Approximately what percentage of all 2500 points do you see?
(b) What are the very first points you see?
(c) What is the first point you see that *doesn't* have y-coordinate 1?
(d) What point do you see exactly halfway along this panning? By this we mean in terms of points seen, not in terms of angle.
(e) What point is halfway through the "first half" of the panning? Can you explain why? You might want to consider some simpler cases first. Again, this is in terms of points seen, not the point at a 22.5° angle.

10. Find the *variance* and *standard deviation* for the number of heads tossed when rolling

(a) ... four dice.
(b) ... five dice.
(c) ... nine dice.
(d) ... n dice.

11. You spin a wheel with the numbers 25, 50, 75, 100 on it. Find the *variance* and *standard deviation* for the total score from spinning the wheel

Go on, spin it! You know you want to. Just be sure to make appropriate "boop boop boop" noises as it goes around.

(a) ... once.
(b) ... twice.
(c) ... four times.
(d) ... 100 times. (Guess?)

12. Find the probability that a positive integer chosen at random is *square free*; that is, it has no factors greater than 1 that are perfect squares.

Tough Stuff.

13. Pick three positive integers. Find the probability that they do not *all* share a common factor greater than 1.

For example, the set $\{15, 21, 35\}$ do *not* share a common factor.