

2007.2 Games People Play

Game of the Day: “Bingo Hi-Lo”

Bill is a contestant on a game show. On this show, an integer between 1 and 75 is picked (at random). Then you are asked whether the next number (picked from the *remaining* numbers) will be higher or lower. If you are correct, you’ll be paid \$100 times the number that comes out. Your plan is to make the decision that makes as much money as possible.

For example, if the first number is 25 and you say “Higher”, and the next number is 35, you’d win \$3500. Easy money!

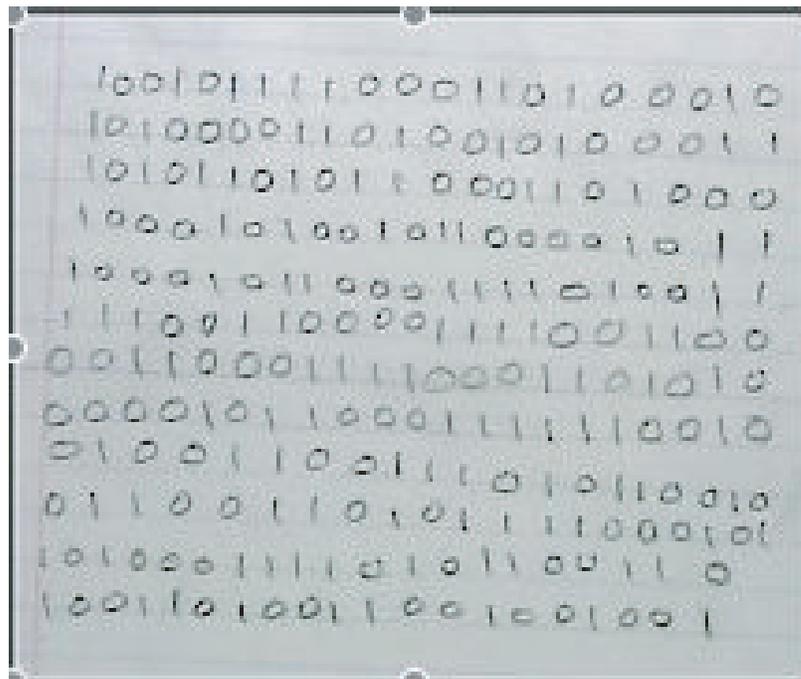
1. (a) Say the first number is 25. What should you do?
- (b) Okay, first number 38. What should you do?
- (c) Okay, first number 42. What should you do?
- (d) So, what’s your overall strategy? State it clearly.

Important Stuff.

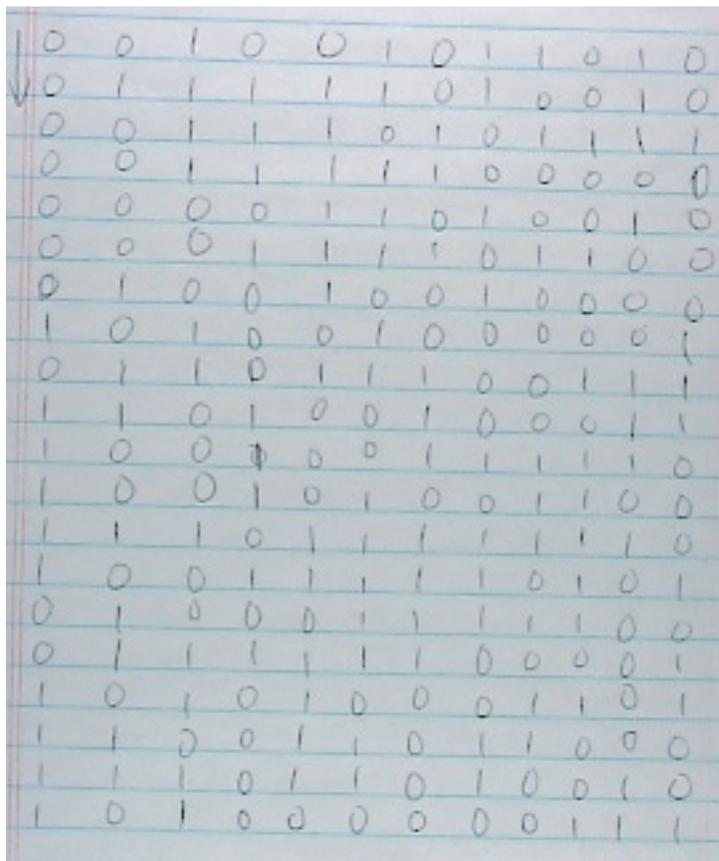
2. Describe, in complete detail, a test you could perform on a set of 240 coin flips that would help you decide whether it is real or fake.
3. Use *your test* on each of these four data sets to decide whether each is real or fake.

They could all be fake; or they could all be real; or . . . If you want more of these, we have them and will happily give you another page of them.

(a)

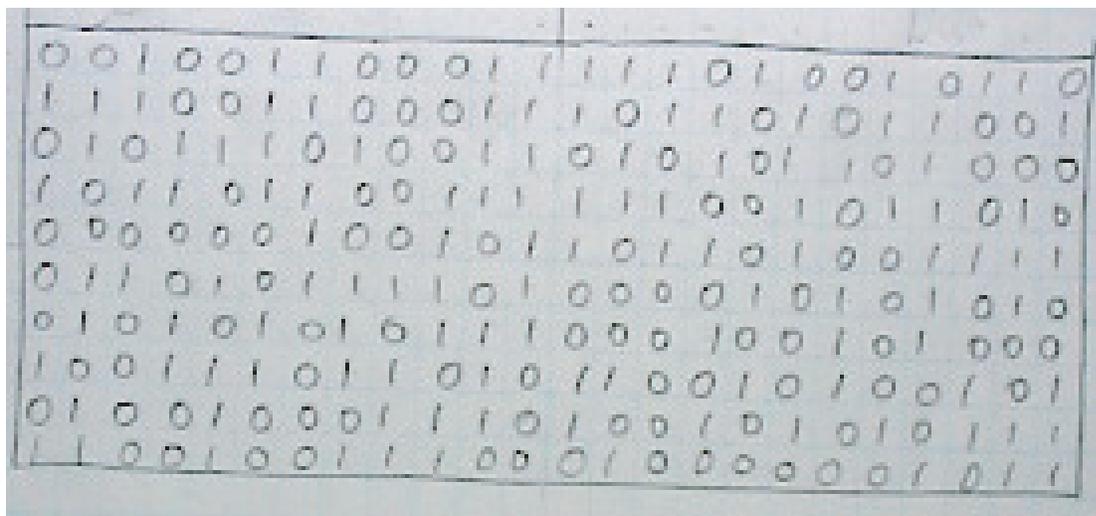


(b)



This one is read vertically; see the arrow. Go down each column. The others are read horizontally.

(c)



(d)

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	1	0	1	1	0	1
2	0	0	1	1	1	0	0	1	0	0	0	0
3	1	1	1	0	0	1	0	0	1	1	1	1
4	0	0	1	1	0	0	0	1	0	1	1	1
5	0	1	1	0	0	1	0	0	0	1	1	0
6	0	1	1	1	1	0	0	1	0	1	1	0
7	1	0	0	0	0	1	0	1	1	1	1	0
8	0	1	1	0	0	1	0	0	0	0	1	1
9	1	0	0	1	0	1	1	1	0	0	0	0
10	1	1	0	1	0	1	1	1	0	0	1	0
11	0	0	1	0	0	0	1	1	0	1	0	0
12	1	1	1	0	1	1	0	0	0	1	1	1
13	1	1	1	1	0	0	1	0	1	0	0	0
14	1	1	0	0	1	0	1	1	1	1	0	0
15	1	0	1	1	0	1	0	0	0	0	1	1
16	0	1	0	0	0	0	1	1	0	1	0	0
17	1	0	1	1	1	0	1	0	0	0	0	0
18	1	0	1	1	1	0	0	1	0	1	1	1
19	0	0	1	0	0	0	1	1	0	1	1	1
20	0	0	1	0	0	0	1	1	0	1	0	0

4. (a) Pick two integers between 1 and 6 (inclusive). Find the probability that the two numbers picked do not share a common factor greater than 1. There is more than one possible correct answer.
- (b) Repeat for picking between 1 and 7, 1 and 8, 1 and 9, 1 and 10.
5. (a) If you flip a fair coin 240 times, how many heads would you expect?
- (b) Take a guess: what is the probability of getting *exactly* this many heads?
6. (a) If you roll a fair die 240 times, how many ones would you expect?
- (b) If you roll a fair die 240 times, what's the expected *sum* of the 240 die rolls?
7. Jocelyn has a piece of paper. She tears it into three equal pieces and hands one piece to Connie, another to Mario, and keeps the third piece for herself.

Don't take more than 10 seconds to guess! Make sure your table has a conversation about this at some point if they haven't already.

She continues to do this; tearing the paper she has left into three equal pieces, handing one piece to Connie, one to Mario, and keeping the third.

- (a) After two tears, how much paper does Jocelyn have left? How much do Connie and Mario each have?
- (b) After three tears?
- (c) After four tears?
- (d) After 10 tears?
- (e) Forever?
- (f) Write two different expressions for the amount of paper Connie has after this is over.

8. Mary has a piece of paper. She tears it into four equal pieces and hands one piece to Manuel, one piece to Kim, one piece to Sandra, and keeps the fourth piece for herself.

She continues to... aw, do we have to write the rest of this? Fine.

- (a) After two tears, how much paper does Mary have left? How much do Manuel, Kim, and Sandra each have?
- (b) After three tears?
- (c) After four tears?
- (d) After 10 tears?
- (e) Forever?
- (f) Write two different expressions for the amount of paper Sandra has after this is over.

Neat Stuff.

9. What's the probability that an integer picked from 1 to n is a perfect square if

- (a) $n = 10$?
- (b) $n = 100$?
- (c) $n = 1000$?
- (d) $n = 10000$?
- (e) What is happening "in the long run" (as n grows larger without bound)?

10. A gambler offers you these two games:

Game 1: You roll a die four times. If you hit a 1 any of the four times, you win.

Game 2: You roll a pair of dice 24 times. If you hit double 1s any of the 24 times, you win.

Aside from the length-of-game issue, which of these games would you rather play to win? Or are they the same?

11. The new toy craze is Mega Men, where kids buy a Mega Man in a box without looking to see which one it is, then open it up when they get home. There are ten toys in all, each equally likely when you buy a box. If you collect all ten, you can make Ultra Mega Mega Man!

On average, how many boxes will you have to buy for your kid before he can finally collect them all?

12. You want to roll 15 as the sum of dice. How many dice do you pick to max your chances?

Sadly, eBay is not an option, since the only cool Mega Men are the ones still in their original packaging.

13. Okay, so there's this game. You get 1 point every time you flip heads. But, anytime you flip tails you're in "danger". If you flip tails a second time *consecutively*, you "bust" and lose all your points (but continue playing).

So, don't flip tails twice in a row. Otherwise it's all good.

- (a) The game lasts 10 flips. What is the probability that you survive all 10 flips without busting even once?
- (b) What is the average score you could expect after 10 flips?
- (c) What happens in a longer game? Will the average score increase? Is there a limit?

Tough Stuff.

14. Remember Yahtzee? Sure, you remember Yahtzee. You get three rolls and you're looking to get all 5 dice to be the same number. You can "save" dice from one roll to the next. There are other goals, but people really only care about getting the 5-dice Yahtzee.

- (a) Find the probability that if you try for it, you will get a Yahtzee of all 6s in any given turn.
- (b) Tougher: find the probability of getting any Yahtzee by trying for it—that is, you always play toward the nearest available Yahtzee.

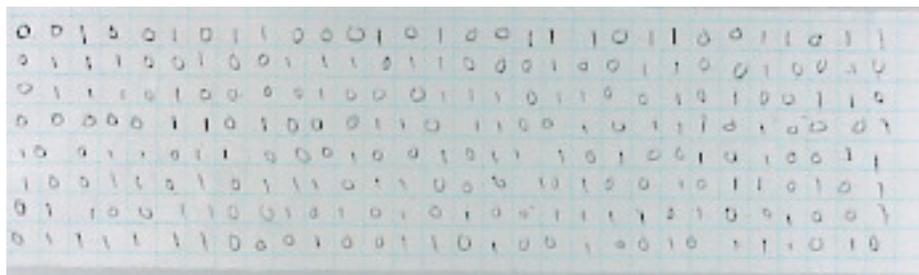
15. Build a data set with at least 5 elements such that if m is the mean and n is the median, then $|m - n|$ is larger than the standard deviation of the set.

Since this is Tough Stuff, we don't have to tell you how to calculate standard deviation.

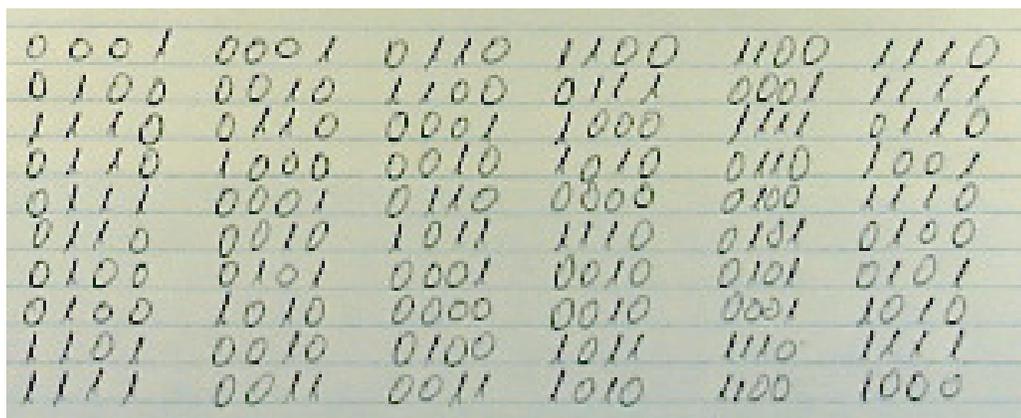
Three More Data Sets

If you want them.

(e)



(f)



(g)

