2007.4 It's Not How You Play The Game, It's How Often You Win Or Lose

Game of the Day: "Two Heads Up"

Time limit: 10 minutes.

Here's three games.

Game 1: Flip two coins. If you get exactly two heads, you win.

Game 2: Flip three coins. If you get exactly two heads, you win.

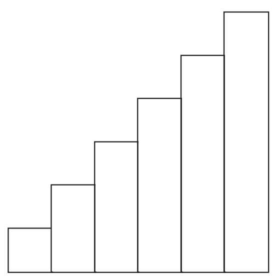
Game 3: Flip four coins. If you get exactly two heads, you win.

1. Which game gives you the highest probability of winning?

Next week is "The Price Is Right" week on Game of the Day. Be prepared.

Important Stuff.

2. (a) Calculate $1 + 2 + 3 + 4 + 5 + 6$.
(b) Here is a geometric shape Peg built from some rods.



If the width and height of each step is 1 cm, find the area of the shape.

3. Calculate the sum of the first 100 positive integers.

4. Take a coordinate grid with $0 \leq x \leq 10, 0 \leq y \leq 10$. Plot all points (x, y) in this range where x and y are both positive with $x \geq y$. Don't do it yet! If x and y share a common factor greater than 1, color them one way; if not, color them another way. Okay, now you can do it. It's safe. It's very safe.

Yes, use green and red! Yes, use pen and pencil! Just make it clear which is which.

5. Let x and y be two integers between 1 and 10, inclusive, with $x \geq y$. Use your graph from problem 4 to determine the probability that any such pair has no common factor greater than 1.

6. Write out F_{10}, the *Farey sequence* of order 10. Yesterday we called this the *Godmother sequence*. Today, John did a five-minute short on this sequence.

If this doesn't make sense, ask Flora, Fauna, or Meriweather. Or just watch Cinderella.

7. Take a coordinate grid with $0 \le x \le 10, 0 \le y \le 10$. Plot all points (x, y) in this range where the fraction $\frac{y}{x}$ is in the Farey sequence. Hmmm?

For example, you'd plot the point $(5,3)$ since $\frac{3}{5}$ is in the Farey sequence. Don't plot $(3,5)$, though.

8. Use your graphs to complete this table. Here, "relatively prime pairs" refers to the pairs plotted in problem 4 that have no common factors greater than 1. Some of the numbers have been included to aid you.

n	Number of elements in F_n	Relatively Prime Pairs
1	2	
2		
3		4
4		
5	11	
6		
7		18
8		
9		
10	33	

9. **Calculator skill time.** Here are some explicit directions on how to expand the expression $(h+t)^2$ on the calculators.

- Hit the HOME button then select option 5 to get a new document.
- When it asks you what kind of page you want, select Calculator.
- Type out the word `expand` then the expression you want to expand. There are two sets of parentheses, one for the `expand` function and one for the expression. The exponent button is on the left side two below CTRL. Your calculator line should look like

  `expand((h+t)`^{2}`)`
- Hit the enter key in the bottom right and enjoy the magic.

Use the green buttons: E-X-P...

(a) Expand $(h+t)^2, (h+t)^3$, and $(h+t)^4$.

(b) What does this have to do with the Game of the Day?

You could also hit CTRL then N for a new document. Lots of the typical command letters work the way they're supposed to.

Neat Stuff.

10. Use the expansion of $(h+t)^6$ to find the total number of ways you could flip 4 heads and 2 tails in a sequence of six coin tosses.

11. Pick a data set of 240 coin tosses, and lump them in 80 groups of three. For each group, count the number of heads: it will be zero, one, two, or three.

(a) How many of each category would you expect? There are 80 total groups.

(b) How does your data set compare? What might make you suspect a fake?

12. Find the *number of elements* in F_{30}, the Farey sequence of order 30. Try to find some shortcuts to help your work: your goal is only to find the number of elements, not list them all.

13. (a) Find the fraction with smallest integer denominator (and integer numerator) that is *between* $\frac{7}{17}$ and $\frac{5}{12}$.

(b) If the answer to the last question is $\frac{a}{b}$, find the fraction with smallest integer denominator (and integer numerator) that is *between* $\frac{7}{17}$ and $\frac{a}{b}$.

14. On the hit show Avery's Bingo Night, the bingo machine is filled with balls for the powers of 2:

$$1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048$$

How would the player's strategy in Tuesday's "Game of the Day" be altered by this change?

15. Today's "Game of the Day" asks you to find the best number of coins to flip to get two heads. What about three heads: what's the number of coins that gives you the best chance of flipping exactly three heads?

16. Draw a histogram with the number of coins flipped on the horizontal, and the probability of flipping exactly four heads on the vertical.

17. It's the weekend: surely you've missed some earlier Neat Stuff problems. If you're interested, check them out. They're neat—but remember that we'll make sure to cover anything really important in... well, yeah.

Tough Stuff.

18. The Price is Right game called "Spelling Bee" asks you to pick five cards from a board containing 30 cards:

11 cards say C

11 cards say A

6 cards say R

2 cards say CAR

Actually, the player needs to answer some easy questions about cutlery or MP3 players to earn the five cards, but we'll assume they get them all.

The goal is to spell CAR in any way possible: if you pull either of the 2 "CAR" cards, you win immediately. But you can also win by picking at least one of each of the three categories C, A, R.

(a) What is the probability of winning this game?

(b) Remarkably, someone actually managed to spell CAR **three** times during the same game. How likely was *this*?

Several video clips of this game are available on YouTube.

19. Henri and Tatyana play a very long game. They flip a coin: if it's heads, Henri gets a point. If it's tails, Tatyana gets a point. What makes it such a long game? Well, in order to win, you have to be 20 points ahead of your opponent.

How long, on average, will this game last?

20. A person is standing at the edge of a pool, and they've had one too many. Each step they take, they have a $\frac{1}{3}$ chance of stepping toward the pool, and a $\frac{2}{3}$ chance of stepping away from the pool.

What is the exact probability that they eventually fall into the pool? Note that this probability will be more than $\frac{1}{3}$ since the first step could take them into the deep end.

21. We've chosen not to repeat any problems on this set. So, if you want some other challenges, go back and look at the other Tough Stuff problems. We'd love to see a few people come back next week with a solution to the Yahtzee problem, or a data set where the difference between the mean and median is greater than the standard deviation.