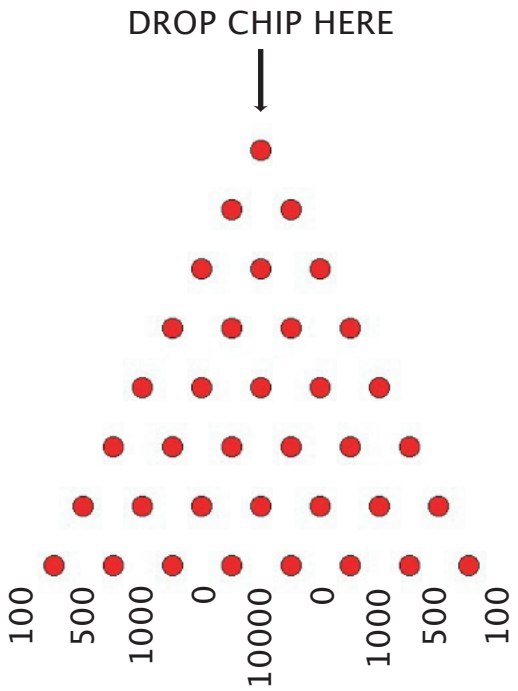
2007.7 Bored Games?

Game of the Day: Plinko!

Plinko is a favorite: all a player has to do is drop a chip, and they can win up to $10,000. Here's a graphic of a simplified version of the Plinko board:

Dude, it's Free Slurpee Day. Go get one. Or several.



Whenever the chip hits a Peg, it has a 50-50 chance of going left or right to the next one. At the bottom, it will fall into one of nine slots with dollar amounts on them, from $0 (d'oh) to $10,000 (woo hoo).

Poor Peg, continually getting beaned by Plinko chips.

1. (a) Describe the relationship between the falling Plinko chip and coin flipping.
(b) Find the probability of falling into the center slot for a $10,000 win.
(c) Find the probability of falling into a $0 slot. Careful: there are two of them.
(d) Find the probability of falling into $1000, $500, $100.
(e) How much, on average, will you win per chip if they let you keep playing this game for a long, long time?
(f) The "real" Plinko is typically played with 5 chips. How much, on average, should players expect to win if they have 5 chips to work with?

Important Stuff.

2. Pick a random integer between 1 and 21, inclusive. What is the probability that this number is . . .

(a) divisible by 3?
(b) divisible by 7?
(c) not divisible by 3?
(d) not divisible by 7?
(e) not divisible by 3, and also not divisible by 7?
(f) divisible by either 3 or 7?

3. Suppose we changed problem 1 to be numbers from 1 to 210, inclusive. Would anything change in the probabilities?

4. Pick any random positive integer. What is the probability that this number is . . .

Ben says, "This is the counting numbers." Bowen says, "Naturally."

(a) divisible by 3?
(b) divisible by 7?
(c) not divisible by 3?
(d) not divisible by 7?
(e) not divisible by 3, and also not divisible by 7?
(f) divisible by either 3 or 7?

What were we thinking, not using 7 and 11 on Free Slurpee Day?

5. Calculator skill time. Here's how to make a scatter plot for the Farey sequence data from yesterday. This one is longer! We will be using this again later in the week(s) ahead.

- Hit the HOME button then select option 5 to get a new document.
- When it asks you what kind of page you want, select Lists & Spreadsheet.
- In the first column type out the numbers 1 through 10.
- In the second column type out the number of elements in the Farey sequence of orders 1 through 10.
- Now go to the *top* of Column A to a small cell right next to the letter A, and type the word `order`. Do the same at the top of Column B and type the word `farey`. This step adds these columns as list data to be used in the other types of page.
- Take a breath for a sec as you flip to the next page.

Find this data in yesterday's Neat Stuff or your earlier notes.

- Hit the HOME button then select option 2 to get a new Graphs & Geometry page. Note that this should *not* be a new document.
- Hit the MENU button then select option 3, "Graph Type" and choice 3, "Scatter Plot." (Hit enter.)
- Hit enter to bring up a list of x variables, and select "order". Then hit tab. Do the same to select "farey" as the y variable. The scatter plot should appear.
- Hit the MENU button then select option 4, "Window" and choice 9, "Zoom - Stat", to fit the window to your data.
- Hit the MENU botton then select option 5, "Trace", to move along the data points.
- Celebrate with your favorite dance.

If you need to move between pages, hit the **ctrl** button then the left or right wheel arrow.

Are we there yet? Are we there yet?

6. Herb stands at the corner of Center St and Main in beautiful downtown Logan, Utah. Assume for the sake of this problem that Logan is a grid of streets (pretty close, actually). Center goes east-west, while Main goes north-south.

Herb flips a coin twice. The first flip tells Herb whether he's going to move north-south or east-west. The second flip tells Herb which direction to go: heads for east or north; tails for west or south. All moves are one block.

Basically all four directions are equally likely with probability $\frac{1}{4}$. After moving a block, Herb decides where to go next.

Poor Herb forgot to pack his tetrahedral dice or Shell Game cups.

(a) After two flips, where could Herb be and with what probability?
(b) After three flips, where could Herb be and with what probability?
(c) After four flips?
(d) Find the probability that Herb is back at the corner of Center and Main after six flips. This one is pretty hard without finding a pattern, so feel free to skip.

7. Ben says that yesterday's problem 6 included a wacky polynomial meant to describe Herb's first walk. Do problem 6 if you haven't. Can you find an even more wacky polynomial that models Herb's second walk?

This comment was partially censored; Ben actually said . . . oh well. Ask him yourself.

Neat Stuff.

8. Why would they put the \$10,000 space in the middle surrounded by two zeros, instead of the other way around?

Think mathematically!

9. Pick an integer between 1 and 900, inclusive. Find the probability that this integer is *square free*; that is, it has no square numbers greater than 1 as factors: no 4, no 9, no 16, and so on.

10. Pick two integers, each between 1 and 30, including duplications and order. There are 900 possible sets, including (21,17) and (17,21). Find the probability that the two numbers do not share a common factor greater than 1.

11. The number of Plinko chips a player actually plays with is 1, plus 4 more they can win at a pricing game. Typically players get 80% of the pricing game questions right. Find the expected payout the show pays for one game of Plinko.

12. Herb moves to Cubertville where the streets have no name. But they run in all directions: east, west, north, south, *up, down*. Herb rolls a die and moves a block; now there are six choices for where he's going to move. But the questions remain:

(a) After two flips, where could Herb be and with what probability?
(b) After three flips, where could Herb be and with what probability?
(c) Find the probability that Herb is back at the corner of Center, Main, and Pogo Blvd (the street that goes up and down) after five flips.

Alright, only these three streets have names.

Tough Stuff.

13. Take the harmonic series and remove all the terms with the number 2 in their denominators:

Sadly this means the fraction $\frac{1}{231}$ is missing.

$$1 + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{19} + \frac{1}{30} + \frac{1}{31} + \frac{1}{33} + \cdots$$

The normal harmonic series diverges (gets larger with no maximum). So, what about this one?

14. Take the harmonic series but consider only terms with prime denominators. Will that converge? If so, to what? If not, can you prove it?