2007.8 Shall We Play A Game?

Game of the Day: "The Big Wheel"

After three contestants on The Price is Right comes the Showcase Show-down, where the player who earns the most moves on. In this simplified version, each player spins twice on a wheel with the numbers 25, 50, 75, 100 on it. For this version, there is no "busting" by going over 100.

1. Find all the possible outcomes of spinning this wheel twice, and the probability that each occurs.

2. What is the average number of points Mary could expect to earn in one spin? two spins?

Mary gives it a good spin and is proud of how it keeps turnin'.

3. Expand this:

$$\left(x^{25} + x^{50} + x^{75} + x^{100}\right)^2$$

What's going on with the exponents and coefficients?

Important Stuff.

4. (a) Find the probability of flipping exactly 4 heads in 8 coin flips.

(b) Use your mad Plinko skillz to answer this question from Day 1 without guessing:

What is the probability of getting *exactly* 120 heads in 240 coin flips?

This is the probability of paying Amy $10,000 . . .

5. Let x and y be integers from 1 to 15, inclusive. There are 225 possible ordered pairs (x, y). For each (x, y), plot it if the greatest common factor of x and y is 1.

We have provided grids for these in the back if you need them. Need more? We got more. All part of our attempts to achieve plentiful sufficiency.

6. *On a separate grid* and *in a different color* plot all the (x, y) where the greatest common factor is 2. Now stare at the grids for a bit.

7. Pick a random integer between 1 and 52, inclusive. What is the probability that this number is . . .

(a) divisible by 4?
(b) divisible by 13?
(c) not divisible by 4?
(d) not divisible by 13?
(e) not divisible by 4, and also not divisible by 13?

Would this better be called indivisible? With liberty and justice for all?

(f) divisible by either 4 or 13?

8. **Calculator skill time.** This builds on yesterday's sketch, and shows how to plot a best-fit function to data, sometimes called a *regression*. Take a deep breath . . .

- Completely complete yesterday's problem 5. You should now have two pages: a spreadsheet page with the Farey sequence data, and a graphing page with the scatter plot.
- Go back to the spreadsheet page by hitting the **ctrl** button then the left wheel arrow.
- Position the cursor in cell D1 of the spreadsheet.
- Hit the MENU button then select option 4, "Statistics", then option 1, "Stat Calculations . . . ", then choose either option 6 or option A for a quadratic or exponential fit.
- A dialog box should come up. For "X List" choose `count`, then hit the TAB button. For "Y List" choose `farey`. Then hit enter twice and watch the magic. The spreadsheet page should now contain the coefficients of the fit in its columns, and the variable `f1` should now store the fit equation.
- Go back to the graphing page by hitting the **ctrl** button then the right wheel arrow.
- Hit the MENU button, then option 3, "Graph Type", then choice 1, "Function".
- Hit the up wheel arrow once to select function `f1` if it isn't already. You should see the regression equation here, perhaps with a few too many digits.
- Hit enter, and the fitting curve will display.

So which is better: a quadratic fit or an exponential one? How could you decide?

This is the godmother of all calculator skill tasks. If you're lucky, you picked up an nSpire with yesterday's task already finished.

Which fit to pick? You decide. Maybe do both.

One great thing about nSpire is that if you went back to add more data to the spreadsheet, everything *immediately updates* to reflect the changes. The list variables, the scatter plot, the fit coefficients, the regression plot... everything!

9. *Variance* is a measure of spread of data: a large variance indicates a wide spread, while a small variance indicates data is tightly packed. In this problem, we're going to show you how to calculate the variance of a data set. Variance is less often called the *mean squared deviation*, but that name describes how to calculate it:

- Find the *deviation* for each element, compared to the mean of the data.
- Take all the deviations and *square* them.
- Find the *mean* of all these values.

Spread 'em.

Consider the data $\{1, 7, 13, 25, 34\}$. The mean is... hm.

(a) What is the mean?

(b) Complete this table to find the variance of the data set. Some of the numbers have been filled in for you.

Psst: try doing this on the nSpire spreadsheet.

Data	Mean	Deviation	Square
1		-15	
7			
13			9
25			
34			324

Variance = **mean of squares** = $\cdots$

(c) The *standard deviation* of a data set is the square root of the variance. So, what is it for this set?

Standard deviation occurs when you teach something not on "the list".

Neat Stuff.

10. (a) Find the variance and standard deviation for the data $\{25, 50, 75, 100\}$ as seen in today's Game of the Day.

You calculated the mean for one and two spins in problem 2.

(b) Find the variance and standard deviation for the data set from *two* spins of the wheel. Note that duplicates *should* be included: there are 3 ways to make 100, and count them all.

(c) Notice anything interesting?

"Yes" is not a complete answer here.

11. The real "Big Wheel" has twenty numbers from 5 to 100, in increments of 5. Also, you bust if you go over 100 combined from two spins.

(a) Find a polynomial that could be used to represent the wheel. No, you don't have to write it all out.

(b) Hey, try this on the calculator:

Bah, I guess you might have to expand it too.

$$\frac{x^{105} - x^5}{x^5 - 1}$$

(c) Find the probability that you bust if you spin twice, even including the idiotic move of spinning again if your first spin was 100.

12. Pick a card from a deck of 52. What is the probability that this card is . . .

(a) a spade?

(b) an ace?

(c) not a spade?

(d) not an ace?

(e) not a spade, and also not an ace?
(f) either a spade or an ace?
(g) This remind you of anything?

13. Problem 7f is kind of interesting: the probability of an integer being divisible by either 4 or 13 is $\frac{4}{13}$. Does this ever happen again? If so, find the general case.

Tough Stuff.

14. So, the "Big Wheel" problem has a little more to it, in terms of strategy. When you spin the wheel the first time, you actually can stop with what you've got, or take a second spin. The goal is to be the closest to 100 without going over.

But what's interesting is that only the last player has full information: they know exactly what amount they need to beat, so it's clear when they should stop or keep going. But the second player doesn't have this.

The tough part is reading this all the way through. There is a lot of cognitive demand here. But hey, it's Tough Stuff, deal with it.

Suppose the first player busted out, and the second player's first spin comes up 50 (on the real wheel that goes from 5 to 100). Should they keep going, or stop? Neither option is really desirable, but which is better? Find a cutoff that says the second player should stop if they spin X or above, and keep going below that.

An even more challenging question is to analyze the first player's decision, since the later players could each beat them. Say the first player's first spin comes up 60. Should they keep going, or stop? If they stop they're pretty likely to get beat by someone, but if they keep going, they are more likely to bust than to improve. Find a cutoff that says the first player should stop if they spin X or above, and keep going below that.

15. There is a continuous analog to the "Big Wheel.", where any real number between 0 and 100 is equally likely to be "spun". The rules remain the same: go over 100 and you're out, and only the best score is a winner.

Develop strategies for the second and first players to determine their first spin cutoffs. You will need some calculus techniques to attack this problem.

15-by-15 grids

Graph paper also a good idea.