2007.9 Big Game Hunter

Game of the Day: "Golden Road"

Golden Road is the richest game available on The Price is Right. Players start given (for free!) a small prize, and try to win bigger and bigger prizes.

Consider a game of Golden Road with three possible outcomes:

- One-fifth of the time, the player wins a lousy spatula worth $5.
- Two-fifths of the time, the player wins a lovely designer rug worth $500.
- Two-fifths of the time, the player wins the big prize, a trip to Banff worth $6,000.

If your actual Golden Road game ended with only a $6,000 prize, you'd be ticked. Most large Golden Road prizes are over $50,000.

1. Find the average payout for this game in the long run.

2. *Variance* is a measure of spread of data: a large variance indicates a wide spread, while a small variance indicates data is tightly packed. Variance is less often called the *mean squared deviation*, but that name describes how to calculate it:

As a spread, you might say "I can't believe it's not variance!" But really it is.

- Find the *deviation* for each element, compared to the mean of the data. Deviation can be positive or negative.
- Take all the deviations and *square* them.
- Find the *mean* of all these values.

So, let's calculate the variance for one play of "Golden Road".

(a) Complete this table to find the variance for one play. Some of the numbers have been filled in for you. The "Mean" column will contain the value you found in problem 1.

Psst: try doing this on the nSpire spreadsheet. The "Mean" column will contain the same number each time.

Data	Mean	Deviation	Square
5		−2596	
500			
500			4414201
6000			
6000			

Variance = **mean of squares** = $\cdots$

(b) Wow, that variance sure seems big. The *standard deviation* of a data set is the square root of the variance. So, what is the standard deviation for one play of "Golden Road"?

Standard deviation occurs when you teach something not on "the list".

Important Stuff.

3. Peter plays a game with five outcomes, but only remembers a few things about its probabilities. One of the five outcomes is the most likely of all five, so he calls its probability p. Two of the four other outcomes have probability two-thirds of p, and the remaining two outcomes are even less likely, only one-sixth of p.

Peter wants to calculate p. What is it? He's listed all five outcomes.

It's unclear whether Peter named the variable after himself.

If you're not sure where to start, try a value for p and see how it goes. Eventually, set up an equation to solve for p.

4. Work with your tablemates to fill out four transparencies, each of which is a 15-by-15 grid.

- Using the **blue** pen on the first transparency, plot all ordered pairs (x, y) with $1 \le x, y \le 15$ such that x and y have no common factors greater than 1. (This was problem 5 on yesterday's set.)
- Using the **red** pen on the second transparency, plot all ordered pairs (x, y) with $1 \le x, y \le 15$ such that the greatest common factor between x and y is exactly 2.
- Using the **green** pen on the third transparency, plot all ordered pairs (x, y) with $1 \le x, y \le 15$ such that the greatest common factor between x and y is exactly 3.
- Using the **black** pen on the fourth transparency, plot all ordered pairs (x, y) with $1 \le x, y \le 15$ such that the greatest common factor between x and y is exactly 4.

5. Now pile all the transparencies together one atop another. What do you notice? Can you explain this?

6. Count the number of dots on the blue and red transparencies. Roughly how many times more blue dots are there than red dots?

7. This data comes from a 240-by-240 grid instead of a 15-by-15 grid. It gives more information about long-term trends.

We thought about having you fill out the 240-by-240 grid and count it up, but it's only a two-hour class.

Color	Dots (out of 57,600)
blue	35,087
red	8,771
green	3,931
black	2,203

(a) Roughly how many times more blue dots are there than red dots?
(b) Roughly how many times more blue dots are there than green dots?
(c) Roughly how many times more blue dots are there than black dots?

8. Check out the zone on the blue transparency with x and y between 1 and 7. See it anywhere else? Is anything like this going on for other colors? Can you use this to explain the patterns in problem 7?

Check, check, check . . . check it out!

They say if you stare at these long enough, you'll see a sailboat or a dolphin.

9. Pick any point (x, y) in a *very* large grid. Let the probability that (x, y) is a blue dot be p.

(a) Give an approximate value for p based on the data in exercise 7.
(b) If the probability of picking a blue dot is p, what is the probability of picking a red dot?
(c) If the probability of picking a blue dot is p, what is the probability of picking a green dot?
(d) Suppose this went on forever: what is the probability, compared to p, that the pair (x, y) will have greatest common factor n?

10. Let p be as in exercise 9. Explain why

$$p + \frac{1}{4}p + \frac{1}{9}p + \frac{1}{16}p + \cdots = 1$$

then use this to write an expression for the value of p.

Neat Stuff.

11. Calculate each of these to six decimal places:

(a)

$$\frac{1}{\sum_{n=1}^{5} \frac{1}{n^2}}$$

(b)

$$\frac{1}{\sum_{n=1}^{25} \frac{1}{n^2}}$$

(c)

$$\frac{1}{\sum_{n=1}^{100} \frac{1}{n^2}}$$

These are the results from problem 10, cut off after 5 terms, 25 terms, and so on. Hit the blue **ctrl** button before entering if you'd like an approximate answer instead of an exact one.

Apologies for a slight technical glitch. Each denominator should look more like

$$\sum_{n=1}^{5} \frac{1}{n^2}$$

(d)
$$\frac{1}{\sum_{n=1}^{1000} \frac{1}{n^2}}$$

(e)
$$\frac{1}{\sum_{n=1}^{10000} \frac{1}{n^2}}$$

What do you notice about the accuracy here?

12. You remember Herb's one-dimensional walk, east-west along Center St? Of course you do! If not, go back and work it out.

(a) Find the *variance* and *standard deviation* for how far away from the start Herb is after two steps.

(b) Find the *variance* and *standard deviation* for how far away from the start Herb is after three steps. Include all eight possible ways.

(c) Repeat for four steps. Wow, that works out nicely for four steps, doesn't it?

(d) What happens in general?

After two steps, Herb is either 0 or 2 blocks from the start.

13. Repeat problem 12 for the two-dimensional case where Herb goes north, south, east, or west. What happens now? Consider Herb's distance from his starting point in Euclidean terms—so, after two steps, he is either 0, 2, or $\sqrt{2}$ blocks from where he started.

This is a pretty amazing result if you ask me. But I guess you didn't! Still, pretty cool.

14. What about Plinko from the top of a tetrahedron with 6 rows? How would this work, and what numbers would come from the Pascalization? What numbers appear at the bottom of the tetrahedron? What numbers appear on the lateral faces?

15. Expand this using the nSpire:

$$(a+b+c)^6$$

What's up with that?

Bonus problem. If train A leaves Chicago at 50 mph, and train B leaves Denver at 60 mph, what is the probability that you'll get this problem right?

Tough Stuff.

16. Repeat problem 12 for the three-dimensional case where Herb goes north, south, east, west, up, or down. What happens now? Again, consider Herb's distance from his starting point in Euclidean terms—after three steps, he could be 1, 3, $\sqrt{5}$, or $\sqrt{3}$ units from the start.

15-by-15 grid

Please use these transparencies, not graph paper. You'll see something interesting! Use blue for 1, red for 2, green for 3, black for 4.

1,2,3,4, do this or we'll sing some more!