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1 *Dot Your I's...*

Hey, welcome to the class. We know you'll learn a lot of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How did others at your table think about this question?
- **Respect everyone's views.** Remember that you have something to learn from everyone else. Remember that everyone works at a different pace.
- **Teach only if you have to.** You may feel the temptation to teach others at your table. Fight it! We don't mean you should ignore your tablemates but give everyone the chance to discover. If you think it's a good time to teach your tablemates about the Law of Cosines, think again: the problems should lead to the appropriate mathematics rather than requiring it. The same goes for technology: the problems should lead to the appropriate use of technology rather than requiring it.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff, and maybe other stuff sometimes. Check out Important Stuff first. All the mathematics that is central to the course can be found and developed in the Important Stuff. *That's* why it's Important Stuff. Everything else is just neat or tough. If you didn't get through the Important Stuff, we probably noticed... and that question will be seen again soon. Each problem set is based on what happened in the previous set, and what happened in the previous *class*.

One question from a previous year turned out to be an unsolved conjecture. Nobody got that one.

On Day 3, go back and read these again.

Will you remember? Maybe!

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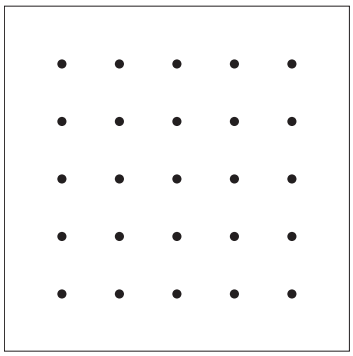
Important Stuff

Come on, you can do it!

Picture a piece of graph paper. Now picture a dot at each intersection. We'll call this *square dot paper*. A 5-by-5 piece of square dot paper would have five dots in each direction – also known as a "geoboard". But the dot paper can be any size, really. We'll say that the distance from a dot to its nearest neighbor is 1.



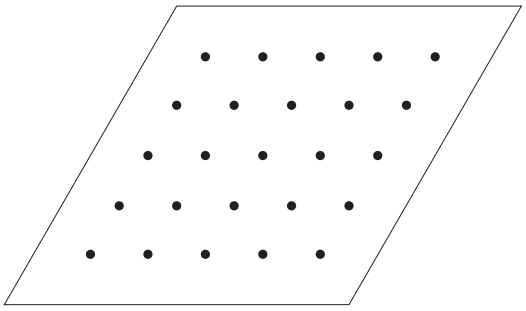
Segments drawn on square dot paper must start and end at dots, but can be horizontal, vertical, or diagonal at any angle.

PROBLEM

On a 6-by-6 piece of square dot paper, what lengths of segments are possible?

Stuff in boxes is more important than other Important Stuff!

There's another type of dot paper, made from a grid of equilateral triangles instead of squares. This is usually called *isometric dot paper*. A 5-by-5 piece of isometric dot paper looks like a "squished" version of the square dot paper, but still has 25 dots. We'll say that the distance from a dot to its nearest neighbor is still 1.



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As before, segments drawn on isometric dot paper must start and end at dots, but can be horizontal, vertical, or diagonal at any angle.

PROBLEM

On a 6-by-6 piece of isometric dot paper, what lengths of segments are possible?

Neat Stuff

What, you're done? Here are some good questions to think about.

1. What happens on larger pieces of dot paper? Make a conjecture before trying.

2. How many *different* lengths are possible on a piece of n-by-n square dot paper? isometric?

3. What kinds of numbers can be distances on square dot paper? isometric? Be as specific as possible.

Tough Stuff

4. Find a non-right triangle with integer side lengths that can be drawn on square dot paper, or prove that no such triangle exists.

5. Find a scalene triangle with integer side lengths that can be drawn on isometric dot paper, or prove that no such triangle exists.

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