No bees, please!

2 *Square Pegs*

Important Stuff

PROBLEM

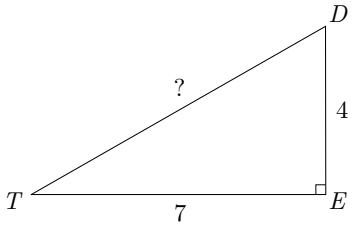
What numbers n can be written in the form $n = x^2 + y^2$? Use the table below to help you look for some patterns. There's more than one!

Let x and y be integers.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96

1. Write each prime as $n = x^2 + y^2$, or determine that it's impossible.
   (a) 101
   (b) 127
   (c) 419
   (d) 421
   (e) 10009

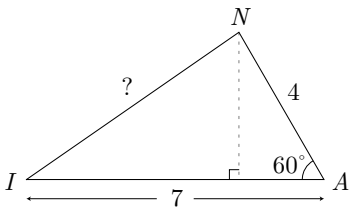
2. Right triangle TED has leg lengths of 7 and 4 as shown.



Find the exact length of the third side.

3. Triangle IAN has two sides with lengths 7 and 4, and a $60°$ angle as shown. Read the side note.

This question should use trigoNOmetry. As in, don't use that.



Find the exact length of the third side.

4. Find all solutions to each equation. Yes, you have to do all six parts.

(a) $x^2 - 12x + 32 = 0$
(b) $x^2 - 12x + 33 = 0$
(c) $x^2 - 12x + 34 = 0$
(d) $x^2 - 12x + 35 = 0$
(e) $x^2 - 12x + 36 = 0$
(f) $x^2 - 12x + 37 = 0$

5. A rectangle has perimeter 24 and area 33. What are its side lengths?

6. Did you finish finding all of the possible segment lengths on a 6-by-6 piece of isometric dot paper? If not, please do that now.

Don't blame us if this takes you a long time. Bree said it wouldn't. She can say that because it's her birthday today.

7. Yesterday we found a segment on square dot paper that *isn't* horizontal or vertical, but still has integer length. Find some others, and describe how you could find more.

Neat Stuff

8. Describe some ways to find Pythagorean triples.

Here are some *Pythagorean triples*: $(3, 4, 5)$, $(5, 12, 13)$ and $(21, 20, 29)$.

9. Are there any other right triangles with integer leg lengths and the same hypotenuse length as triangle TED?

10. Modify triangle IAN from problem 3 so that it has the same $m\angle A = 60°$ and length IN, but different (integer) lengths for IA and/or AN.

11. What numbers n can be written in the form $n = x^2 - y^2$?

12. What numbers n can be written in the form $n = x^2 + y^2 + z^2$?

13. What numbers n can be written in the form $n = x^2 + y^2 + z^2 + w^2$?

14. How many *different* lengths are possible on a piece of n-by-n square dot paper? isometric?

15. What kinds of numbers can be distances on square dot paper? isometric? Be as specific as possible.

Tough Stuff

16. Find a non-right triangle with integer side lengths that can be drawn on square dot paper, or prove that no such triangle exists.

17. Find a scalene triangle with integer side lengths that can be drawn on isometric dot paper, or prove that no such triangle exists.

18. The quadratic equation $x^2 - 10x + 22 = 0$ has two roots.

(a) Find a quadratic whose roots are the *squares* of the roots of $x^2 - 10x + 22 = 0$.

(b) Find a quadratic whose roots are the nth powers of the roots of $x^2 - 10x + 22 = 0$.

19. Find all integer solutions to this system of equations.

There are probably more than you think.

$$\begin{aligned} a + b &= cd \\ c + d &= ab \end{aligned}$$

No bees, please!

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