Did you remember to reread the first page of the problems from Day 1?

3 *Triangular Pegs*

Important Stuff

PROBLEM

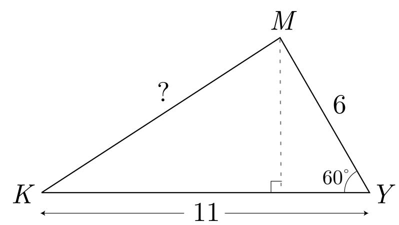
What numbers n can be written in the form $n = x^2 + y^2 - xy$? Use the table below to help you look for some patterns. There's more than one!

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96

It's vuja de, the feeling that none of this has ever happened before.

1. Write down all the possible segment lengths on a 6-by-6 piece of isometric dot paper from smallest to largest. Don't use a CALculator to put them in order.

2. Triangle KYM has two sides with lengths 11 and 6, and a 60° angle as shown.



(a) Find the exact length of the third side.

Two be. There is no question.

3. Simplify these.

(a) $(3 + 2\sqrt{2})(3 - 2\sqrt{2})$
(b) $(10 - 7\sqrt{2})(10 + 7\sqrt{2})$
(c) $(a + b\sqrt{2})(a - b\sqrt{2})$

Don't ask us why these are here. But, $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$ are *conjugates* of each other.

Did you remember to reread the first page of the problems from Day 1?

4. Figure out a way to rewrite these without using fractions.

(a) $\dfrac{7+4\sqrt{2}}{3+2\sqrt{2}}$

(b) $\dfrac{8-5\sqrt{2}}{10-7\sqrt{2}}$

(c) $\dfrac{8-5\sqrt{2}}{5+3\sqrt{2}}$

Ask Sendhil what *rat the den* means.

Didn't we just work with square roots of 2 in the last problem? Sheesh.

5. Let $a = 2+3i$ and $b = 7-4i$. Simplify these.

(a) $a+b$
(b) $a-b$
(c) ab (the product)

Do this like regular algebra, except whenever you see i^2, replace it by -1. That's the big property of i, it's the square root of -1.

6. Repeat problem 5 with $a = 7+4i$ and $b = 7-4i$.

7. Simplify these.

(a) $(3+2i)(3-2i)$
(b) $(2-i)(2+i)$
(c) $(a+bi)(a-bi)$

$3+2i$ is the conjugate of $3-2i$ and vice versa.
$3+2i$ was the conjugate.
$3+2i$ will be the conjugate.
$3+2i$ has been the conjugate.
$3+2i$'s the conjugate.
$3+2i$ is all like the conjugate.
$3+2i$ can has conjugate?

8. Figure out a way to rewrite these without using fractions.

(a) $\dfrac{13+26i}{3+2i}$

(b) $\dfrac{18-i}{2+i}$

(c) $\dfrac{18-i}{7-4i}$

Neat Stuff

9. Describe some ways to find Pythagorean triples.

10. What kinds of numbers can be distances on square dot paper? On isometric dot paper?

11. Modify triangle KYM from problem 2 so that it has the same $m\angle Y = 60^\circ$ and length KM, but different (integer) lengths for KY and/or YM.

12. How many different squares can you draw on an n-by-n piece of square dot paper using only horizontal and vertical segments?

Did you remember to reread the first page of the problems from Day 1?

13. Put the following points in order of their distance from the origin, from closest to farthest.
(a) $R = (0, 5)$
(b) $A = (-3, 4)$
(c) $U = (2, 5)$
(d) $L = (-4, -4)$

14. Evaluate the function N for each of these numbers and order them from their lowest to highest N-value. Read the note on the right.
(a) $m = 0 + 5i$
(b) $a = -3 + 4i$
(c) $r = 2 + 5i$
(d) $y = -4 - 4i$

The function N takes a number and multiplies it by its conjugate.
$N(3+2i) = (3+2i)(3-2i) = 13$
$N(2-i) = (2-i)(2+i) = 5$
This function is sometimes called the "norm," but that term is not used consistently so we won't use it.

15. In class, we conjectured that any number that is one more than a multiple of 12 can be written as the sum of two squares (of integers). Does this always work?

16. In class, we conjectured that any number that is one less than a multiple of 4 cannot be written as the sum of two squares (of integers). Does this always work?

17. Write each prime as $n = x^2 + y^2 - xy$, where x and y are integers, or determine that it's impossible.
(a) 101
(b) 127
(c) 419
(d) 421
(e) 10009

Tough Stuff

18. Let (x, y) be a point on the unit circle. If you walk along the circle from $(1, 0)$ to (x, y), then walk that same distance farther along the circle, where will you be?

This question should use trigoNOmetry. As in, don't use that.

19. What prime numbers are squares in mod 17? (Include primes that are larger than 17.) What primes p make 17 a perfect square in mod p?

20. Find all integer solutions to this system of equations.

There are probably more than you think.

$$\begin{aligned} a + b &= cd \\ c + d &= ab \end{aligned}$$

This page intentionally left blank.