Ninja cowboy bear!! Yay!

8 *times i equals infinity*

Important Stuff

PROBLEM

Let $w = \dfrac{-1 + i\sqrt{3}}{2}$. Calculate these.

(a) w^2 (b) $\overline{w}$ (c) $-1 - w$
(d) ww^2 (e) $w\overline{w}$
(f) $w + w^2$ (g) $w + \overline{w}$

Take a few moments to write a summary of everything that you know about w.

Did you watch wwE Raw last night?

1. Expand $(9+4w)(9+4\overline{w})$ and simplify your answer as much as you can.

No, not like this:

$(9{+}4w)(9{+}4\overline{w})$

2. Plot the following points on a sheet of isometric dot paper. How far is each point away from the origin?
(a) $M = [\rightarrow 9, \nwarrow 4]$
(b) $A = [\rightarrow 4, \nwarrow 9]$
(c) $R = [\rightarrow 9, \nwarrow -4]$
(d) $C = [\rightarrow 7, \nwarrow 9]$
(e) $Y = [\rightarrow -7, \nwarrow -9]$

Trust your intuition on this one. $[\rightarrow 9, \nwarrow -4]$ is the same as $[\rightarrow 9, \searrow 4]$

3. Plot each of the following points in a complex plane. You may need to convert some numbers to decimals to plot them accurately.
(a) $9 + 4w$
(b) $4 + 9w$
(c) $9 - 4w$
(d) $7 + 9w$
(e) $-7 - 9w$

Ninja cowboy bear!! Yay!

4. Expand these. Which of them work out nicely?
(a) $(2+3w)(2-3w)$
(b) $(2+3w)(2+3\overline{w})$
(c) $(7+4w)(7-4w)$
(d) $(7+4w)(7+4\overline{w})$

these.

For today's problem set, anytime a problem involves w, you can leave w in your answers, but try not to have w^2 and $\overline{w}$.

5. (a) Calculate $N(7+4w)$.
(b) Calculate $N(5+2i\sqrt{3})$.
(c) How are $7+4w$ and $5+2i\sqrt{3}$ related?

6. Calculate the N-value of the following numbers.
(a) $9+4w$
(b) $4+9w$
(c) $9-4w$
(d) $7+9w$
(e) $-7-9w$

Reminder: we defined N to be a function takes a number and multiplies it by its conjugate.

Neat Stuff

7. Let $p = a+bi$ and $q = c+di$, with a, b, c and d being arbitrary real numbers. Prove or disprove these statements.
(a) $\overline{pq} = \overline{p}\,\overline{q}$
(b) $N(pq) = N(p)N(q)$
(c) $N(p^2) = N(p)^2$

This problem is *sneat*. In other words, it's super neat. So that means try it first. Look at yesterday's problem set for more of this problem.

8. Find two numbers whose sum is -1 and product is 1.

9. Factor $33+4i$ as much as you can.

10. Remember how we defined N to be a function that takes a number and multiplies it by its conjugate? In $\sqrt{2}$-land, the conjugate of $a+b\sqrt{2}$ is $a-b\sqrt{2}$, so $N(a+b\sqrt{2}) = (a+b\sqrt{2})(a-b\sqrt{2}) = a^2-2b^2$. Calculate these.
(a) $N(5+2\sqrt{2})$
(b) $N(5-2\sqrt{2})$
(c) $(5+2\sqrt{2})^2$
(d) N(your answer to part c)
(e) $N(4+3\sqrt{2})$
(f) $(5+2\sqrt{2})(4+3\sqrt{2})$
(g) N(your answer to part f)

11. If $a^2+b^2=c^2$, why is the point $\left(\frac{a}{c},\frac{b}{c}\right)$ on the unit circle?

12. What does the graph of $x^2+y^2-xy=1$ look like?

13. Show that if $\left(\frac{a}{c}, \frac{b}{c}\right)$ is a point on the graph of $x^2+y^2-xy=1$ then $a^2+b^2-ab=c^2$.

14. Find the intersection(s) of the unit circle $x^2+y^2=1$ and these lines.
(a) $y = 3x - 1$
(b) $5x - 3y = 3$
(c) $7y + 7 = 9x$

15. Find the intersection(s) of the curve $x^2+y^2-xy=1$ and these lines.
(a) $y = 3x - 1$
(b) $5x - 3y = 3$
(c) $7y + 7 = 9x$

16. Find a method that can produce infinitely many triples (a, b, c) with no common factors such that a triangle with sides a, b, and c can be drawn on isometric dot paper.

Drawn on isometric dot paper = two sides must lie along rows of dots and all three corners must land on dots. You might want to find one or two such triangles first!

17. How many different peg-to-peg squares are there on an $n \times n$ piece of square dot paper?

Oh noes: starting with 3×3, there are bonus squares!

18. How many different peg-to-peg squares are there on an $n \times n$ piece of isometric dot paper?

19. How many different peg-to-peg equilateral triangles are there on an $n \times n$ piece of isometric dot paper?

Bonus triangles!!!

20. Find all points on the isometric dot plane, $[\rightarrow x, \nwarrow y]$, with x and y integers, that are 19 units away from the origin. What about 67?

Did you know that Will Ferrell was born on July 16, 1967? Me neither.

21. Let $w = \dfrac{-1 + i\sqrt{3}}{2}$. Use exact arithmetic to calculate these.
(a) w^{-1}
(b) $(\overline{w})^{-1}$

Tough Stuff

22. Find a distance so that more than 24 points on a isometric dot plane are that distance away from the origin.

23. Find a distance so that more than 24 points on a square dot plane are that distance away from the origin.

Ninja cowboy bear!! Yay!

24. Find a distance so that more than 24 points on *both* square and isometric dot planes are that distance away from the origin.

25. Are there any other kinds of dot paper that would work for the problems over the last two weeks?