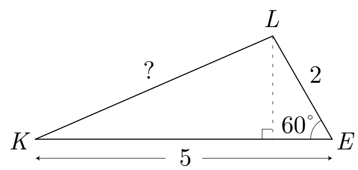
This movie was re-released as "i Am Legend(re)."

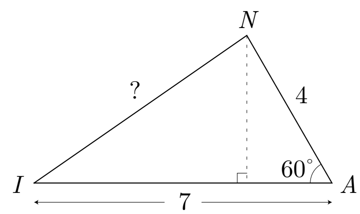
9 *The w (Ra)Man(ujan)*

Important Stuff

1. Triangle KEL has two sides with lengths 5 and 2, and a 60° angle. Triangle IAN has two sides with lengths 7 and 4, and a 60° angle.

These two triangles aren't drawn using the same scale, but their shapes are correct.





Find the exact length of the missing sides.

PROBLEM

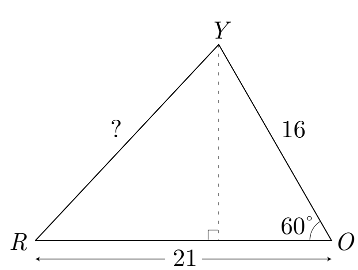
For each of these numbers, square it, then calculate the N-value of the original number.

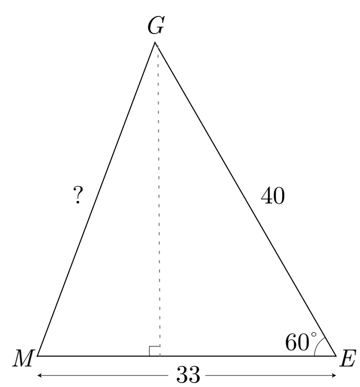
(a) $5 + 2w$ (b) $7 + 4w$
(c) $3 + w$ (d) $4 + w$
(e) $7 + w$ (f) $9 + 2w$

When you square the number, try to write it so it looks like $a + bw$.

2. Triangle ROY has two sides with lengths 21 and 16, and a 60° angle. Triangle MEG has two sides with lengths 33 and 40, and a 60° angle.

$\triangle ROY$ will from now on be abbreviated as TROY.





Find the exact length of the missing sides.

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3. Recalculate the missing length in TROY in problem 2 if...
(c) $RO = 8$ and $OY = 5$
(d) $RO = 15$ and $OY = 7$
(e) $RO = 48$ and $OY = 13$
(f) $RO = 77$ and $OY = 32$

4. Describe a way to find triangles with integer side lengths that have a 60° angle.

Let's call these integer side lengths *Einstein triples*. Einstein triples relate to triangles with a 60° angle in the same way that Pythagorean triples relate to triangles with a 90° angle.

5. Remember how we defined N to be a function that takes a number and multiplies it by its conjugate? In $\sqrt{2}$-land, the conjugate of $a + b\sqrt{2}$ is $a - b\sqrt{2}$, so $N(5 + 2\sqrt{2}) = (5 + 2\sqrt{2})(5 - 2\sqrt{2}) = 17$. Calculate these.
(a) $N(1 + 2\sqrt{2})$
(b) $N((1 + 2\sqrt{2})^2)$
(c) $N((1 + 2\sqrt{2})^3)$
(d) $N((1 + 2\sqrt{2})^4)$
(e) $N((1 + 2\sqrt{2})^5)$

Everett,WA+$\sqrt{2}$ =Duluth,MN

Neat Stuff

6. Find integers a and b so that $N(a+bw) = 19$. Find integers c and d so that $N(c+dw) = 21$. Calculate $(a+bw)(c+dw)$, then calculate $N((a + bw)(c + dw))$.

7. Use the results of problem 6 to find an Einstein triple (a, b, c) with $c = 399$.

Like the price of gas, this number goes up daily.

8. Remy stands at the origin and stares at $1 + w$, $(1 + w)^2$, $(1 + w)^3$ and so on. Describe what happens to the powers of $1 + w$ from her perspective: where do they go? how far away (N-value, anyone)?

9. Find all points $[\rightarrow x, \nwarrow y]$ on an isometric dot plane with integers x, y that have $N(x + yw) = 259$.

10. Figure out a way to rewrite these fractions as simply as possible.
(a) $\dfrac{2 + 17w}{7 + 4w}$
(b) $\dfrac{2 + 17w}{2 + 3w}$

Unscramble this:
"hat red net"
Also see problem 4 on Day 8.

11. Extend your method from problem 4 to generate some triangles with integer side lengths that have a 120° angle.

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12. Factor $49 - 8i$ as much as you can.

13. Pick some other integers a and b and use your method from problem 4 to generate some Einstein triples from $a + bw$. You may find that sometimes your side lengths are zero or negative. Refine your method so that this doesn't happen.

14. Does squaring $a+bw$ give you all possible Einstein triples?

15. Find a number n that is the the largest number of exactly four primitive Einstein triples.

16. What's the relationship between $\angle K$ in $\triangle KEL$ and $\angle R$ in TROY? What's the relationship between $\angle I$ in $\triangle IAN$ and $\angle M$ in $\triangle MEG$?

17. Plot the graph of $x^2 + y^2 - xy = 1$ on isometric dot paper.

Tough Stuff

18. Find a distance so that more than 24 points on a isometric dot plane are that distance away from the origin.

19. Find a distance so that more than 24 points on a square dot plane are that distance away from the origin.

20. Find a distance so that more than 24 points on *both* square and isometric dot planes are that distance away from the origin.

21. Are there any other kinds of dot paper that would work for the problems over the last two weeks?

John wants to know about pentagonal dot paper.

22. Does the location of the major axis of $x^2 + y^2 - xy = 1$ have anything to do with the location of w in the complex plane?

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