11 *Fantasy i-land*

Important Stuff

PROBLEM

Today's problem can't fit in this box, so we're calling it the "problem out of the box." We promise it's still fun!

The charts with most the beautiful handwriting will get a surprise.

1. What peg-to-peg distances are possible on a 6-by-6 piece of square dot paper?

Don't take more than three minutes on this problem.

2. Let $a = 2 - i$. Plot these numbers on a complex plane.

$$\begin{array}{rrrrr} (-2+2i)a & (-1+2i)a & 2ia & (1+2i)a & (2+2i)a \\ (-2+i)a & (-1+i)a & ia & (1+i)a & (2+i)a \\ -2a & -1a & 0a & a & 2a \\ (-2-i)a & (-1-i)a & -ia & (1-i)a & (2-i)a \\ (-2-2i)a & (-1-2i)a & -2ia & (1-2i)a & (2-2i)a \end{array}$$

What do you notice?

3. Which Gaussian integers have N-value equal to 1?

A *Gaussian integer* is a complex number $a + bi$ that has integers for a and b. So, -2, i, $4 - i$ and $-5 + 3i$ are Gaussian integers, but $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ is not.

4. Calculate these.
   (c) $1 \cdot 13$
   (a) $i(-13i)$
   (m) $(-1)(-13)$
   (i) $(-i)(13i)$

5. In i-land, 5 is not prime because it factors: $5 = (2+i)(2-i)$. Which of these numbers factor into Gaussian integers?
   (s) 2
   (a) 3
   (n) 7
   (d) 13
   (r) 19
   (a) 23

Only consider Gaussian integer factors with N-values greater than 1. Why? Because the factorizations of 13 in Problem 4 are boring.

Neat Stuff

6. On Part 1 of today's "problem out of the box," use color pencils to color in the circles according to the size of the number. For example, use a different color if the number is between (and including) 1 and 25, between 26 and 50, between 51 and 75, between 76 and 100. Count the number of circles in each category.

Stay within the lines!!

7. Look at the chart from Part 2 of today's "problem out of the box" to see what patterns you can find.

8. If you're still interested in thinking about Pythagorean and Eisenstein triples, go back to the problems on Day 10 that you didn't finish.

9. Prove that if $z = ab$, then $N(z) = N(a) \cdot N(b)$.

10. Use what you know about the N function to factor $16+3i$ into two Gaussian integers.

11. The prime factorization of any (real) positive integer into other positive integers is unique. What about the factorization of Gaussian integers into other Gaussian integers? Consider this factorization example:

$$8+i = (2-i)(3+2i) = (1+2i)(-i)(3+2i) = (1+2i)(2-3i)$$

Do you think the prime factorization of a Gaussian integer into other Gaussian integers is unique? Why or why not? If you think Gaussian integer factorizations are unique, explain what you mean by "unique." If you think they are not unique, describe what the different factorizations for a number have in common.

12. Graph $x^2 - 2y^2 = 1$. What does that have to do with the problem in the box from Day 10?

13. Use the method from the problem in the box on Day 10 to find a good approximation to $\sqrt{11}$.

Come on, you can do better than that!

14. Pick an integer n with $2 \le n \le 12$. On Part 1 of today's "problem out of the box," use color pencils to color in the numbers according to their remainder after dividing by n.

15. Which Gaussian integers are prime?

Seek inspiration from Simon and Garfunkel if the problem in the box is getting you down.

Tough Stuff

16. Find a triangle with integer sides with a $30°$ angle, or prove that none exist.

17. Find three lattice points in $\mathbb{R}^3$ that are integer distances apart. The triangle that is formed cannot be parallel to the xy-, yz-, or xz-planes.

18. This question is about the problem in the box on Day 10. What does the N-value of the starting number have to do with the convergence rate of the fractions?

19. The Price is Right wheel has the amounts ranging from 5 cents to $1.00 in increments of 5 cents. Suppose you get to keep spinning until you go over $1.00. What is the average number of times you will spin the wheel until you go over?

Last year in class we spun this wheel, and the entire class chanted "Beep, beep, beep.. beep...."

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