Santa Teresa Middle School Lesson Study Team Project 2007/08

Team Members:
Castro, Ana P.
Medrano, Corina
Sherk, Constance
Wollard, Kimberly

Santa Teresa Middle School Lesson Study Team Meeting Dates

August	6	Chicago Lesson Study Conference	8:30 – 4:00
	7	Chicago Lesson Study Conference	8:30 – 4:00
	8	Chicago Lesson Study Conference	8:30 – 4:00
	9	Chicago Lesson Study Conference	8:30 – 4:00
	10	Chicago Lesson Study Conference	8:30 – 4:00
	27		2:00 – 3:30
September			
	10		2:00 – 3:30
	18		2:00 – 3:30
	24	Practice Lesson & Debrief	8:30 – 3:30
	27 & 28	Gadsden/L.C. Group	8:30 – 3:30
October			
	22		2:00 – 3:30
	29		2:00 – 3:30
November	8		4:30 – 7:30
	9, 10	Gadsden/L.C. Group	8:30 – 3:30
Jan. 31 & Feb 1		Gadsden/L.C. Group	8:30 – 3:30
February			
	13		2:00 – 3:30
March	1		11:30 – 3:30
April	10		2:00 – 3 ;30
	11		2:00 – 3:30
May	8	Chicago Lesson Study Conference	8:30 – 3:30
	9	Chicago Lesson Study Conference	8:30 – 3:30
	10	Chicago Lesson Study Conference	8:30 – 3:30

GROUP: Santa Teresa Middle School

GRADE LEVEL: 8th

TIME OF LESSON: 9:00 – 10:30

TEACHER NAME: Kim Wollard

ROOM: Library

SCHOOL NAME: Santa Teresa Middle School

SCHOOL ADDRESS: 4800 McNutt Road

Santa Teresa, New Mexico

SCHOOL PHONE: (505) 874-7200

Mathematics Lesson Plan

September 28, 2007
Santa Teresa Middle School
Eighth Grade
Instructor: Kim Wollard
Lesson plan developed by: Corina Medrano,
Kim Wollard, Connie Sherk, Ana Castro

1. Title of the Lesson:

Making Smaller Ballots: Introducing Exponential Decay

2. Goals of the Lesson:

a. Use knowledge of exponential relationships to make tables and graphs and to write equations for exponential decay patterns.
b. Analyze and solve problems involving exponents and exponential decay.

3. Relationship of the Lesson to the Standards

Prior units of study: Variables and Patterns, Moving Straight Ahead, Thinking with Mathematical Models.
Standards:
Strand- Algebra.

- In 7th grade the students were introduced to looking for graphical or symbolic models to describe a pattern.
- They had experience with representing relationships with words, tables, graphs and equations.
- They explored the significance of shapes of graphs and patterns in tables.
- They have worked with attaching meaning to the symbols in a linear equation of the form y = mx + b.
- They have had experience with recognizing the significance of constant additive growth.
- The explored recognizing and describing situations that can be modeled by linear relationships.
- They used exponents to express large and small quantities.

Related post learning standards (topics/objectives)

- Extending the analysis to include all positive real numbers for the domain.
- Using tabular, graphical, and symbolic methods to solve problems that involve exponential functions, such as finding half-life or solving equations of the type $a^x = b$.
- Exploring the significance of shapes of graphs and patterns in tables.
- Extending the experience to include recognition of logarithmic and trigonometric relationships.
- Making sense of the symbols in quadratic relationships expressed and expanded or factored forms.
- Reviewing and extending the analysis of exponential and quadratic functions.
- Analyzing symbolic expressions of trigonometric and logarithmic functions.
- Recognizing the significance of the pattern of change in quadratic relationships.
- Analyzing patterns of change and exponential logarithmic and trigonometric functions.
- Recognizing and describing situations that can be modeled by quadratic functions.
- Extending recognitions to logarithmic and trigonometric functions.
- Applying rules for exponents to interpret more complex algebraic expressions and exponential equations.

8th grade Lesson Standard & Benchmarks

- Strand 2: Benchmark 2 (Algebra): Use symbols, variables, expressions, inequalities, and simple systems of equations to represent problem situations that involve variables or unknown quantities.
- Strand 2: Benchmark 4(Algebra): Use graphs, tables, and algebraic representations to make predictions and solve problems that involve change.
- Strand 2: Benchmark 4 (Algebra): Use appropriate problem solving strategies to solve problems that involve change.
- Strand 2: Benchmark 4 (Algebra): Generalize a pattern of change using algebra and show the relationship among the equations, graph, and table of values.
- Strand 2: Benchmark 4 (Algebra): Recognize the same general pattern of change present in different representations.
- Strand 1: Benchmark 2 (Number and Operations): Use real number properties to perform various computational procedures.
- Strand 2: Benchmark 3(Algebra): Generate different representations to model a specific numerical relationship given one representation of data.
- Strand 1: Benchmark 3(Number and Operations): Formulate algebraic expressions that include real numbers to describe and solve real world problems.

4. Unit Plan

Name of Unit: Growing, Growing, Growing: Exponential Relationships from *The Connected Mathematics Program 2* (2006). Pearson, Prentice Hall: Michigan State University.

Investigation 1: Exponential Growth

- Students explore situations that involve repeated doubling, tripling, and quadrupling. Students are introduced to one of the essential features of many exponential patterns: rapid growth.
- Students make and study tables and graphs for exponential situations, describe the patterns they see, and write equations for them. The students compare linear and exponential patterns of growth.

Investigation 2: Examining Growth Patterns

- Students will focus on exponential relationships with y-intercepts greater than 1. Each problem in the investigation presents information about an exponential pattern in a different form – in a verbal description, in an equation, and as a graph- helping students develop flexibility in moving among representations.

Investigation 3: Growth Factors and Growth Rates

- Students will study non-whole number growth factors other than 1 and relate these growth factors to growth rates. Students will also explore how the growth factor and the initial value affect the growth pattern.

This research lesson focuses on Investigation 4: Exponential decay.

Investigation 5: Patterns with Exponents

- Students will examine patterns among the ones digits of powers and use these patterns to predict ones digits for powers that would be tedious to find directly. They will look for relationships among the numbers written in exponential form. They will use graphing calculators to study the effects of the values of a and b on the graph of $y = a(b^x)$.

5. Instruction of the Lesson

The rationale for teaching this lesson includes the following: Our students struggle with the conceptual understanding of exponents. The topic of exponential growth and decay is a topic in our school that has typically not been addressed in an effective manner.

According to the standards and curriculum the students need to:

- Use knowledge of exponential relationships to make tables and graphs and to write equations for exponential decay patterns.
- Analyze and solve problems involving exponents and exponential decay.
- Recognize patterns of exponential decay in tables, graphs, and equations.
- Use information in a table or graph of an exponential relationship to write and equation.
- Analyze an exponential decay relationship that is represented by an equation and use the equation to make a table and a graph.

Based on observations the students have learned so far:

- Students begin identifying simple patterns as early as kindergarten and continue to work with patterns throughout the elementary years of school. By the end of 7^{th} grade the students have had experience with identifying patterns using tables, graphs and verbal and written descriptions. They have been exposed to the concept of linear relationships through verbal and written, tabular and graphic representations. Students have engaged in classroom discourse and written descriptions expressing their mathematical thinking. Students have been working in collaborative groups throughout their eight years of schooling. The seventh grade curriculum focused heavily on analysis of mathematical data and using that analysis to make inferences and predictions about patterns they encounter.
- The basic goal in Growing, Growing, Growing is for students to learn to recognize situations, data patterns, and graphs that are modeled by exponential equations and to use tables, graphs, and equations to answer questions about exponential patterns. This unit is designed to introduce the topic and to give students a sound, intuitive foundation on which to build later.

The major focus of this lesson:

- The major focus of this lesson is exponential decay. The students will revisit the paper-cutting activity of investigation 1 with a new question in mind: How does the area of a ballot change with each successive cut? The students will make tables, graphs and write equations to show exponential decay patterns. They will analyze and solve problems involving exponents and exponential decay.

The above objective will be accomplished by doing the following:

- By presenting the students with the situation of cutting an 8 x 8 piece of paper in half to create ballots they will explore how the area of the ballots changes. They will collect data to create table to show the relationship of the number of cuts and the area of the ballots. Then they will write an equation to generalize the situation. They will graph their data. Through class discussions of the students' data they will infer how the patterns of change between exponential and decay growth patterns are similar and different.

Steps, Learning Activities *Teacher's Questions and Expected Student Reactions*	*Teacher's Support*	*Points of Evaluation*
Introduction From the textbook: In Problem 1.1, you read about the ballots Chen, the secretary of the Student Government Association, is making for a meeting. Recall that Chen cuts a sheet of paper in half, stacks the two pieces and cuts them in half, stacks the resulting four pieces and cuts them in half, and so on. You investigated the pattern in the number of ballots created by each cut.		
Posing the Problem In this problem, you will look at the pattern in the areas of the ballots. Chen begins making ballots with a paper that has an area of 64 square inches. Your works today is to demonstrate mathematically in as many ways as possible how the area of the ballot changes after each of 10 cuts, and then describe the change in the area after 100 cuts, and finally how would the area change for any number of cuts.		
Anticipated Student Responses *C1: 2 + (3 * 5) [correct]* *C2: 3 * 5 = 15; 2 + 15 = 17* Table: • We predict the students will begin their columns by graphing a table. Most of the students will label the x and y column correctly. Some of the students will indicate area in fractional form. Some of the students will indicate area of ballots in decimal form. • We predict that some students will omit the row for 0 cuts with an area of 64 in.^{2} At the seventh cut they may indicate that the area is .5 or ½. Graph: • We predict that some students will make a graph that is represented by using the values in the y column as their intervals on the y-axis. Equation: • We predict they will come up with the following: A = l x w A = ½ lw A = 64 x ½ A = 64n/2 A = (64/2)(n) A = (2n)(64) $A = 64(1/2)^n$ Something of the form y = mx +b	• If the students did not begin their table with 0 cuts and 64 in.^{2} we will question them through the process of why their table should begin with this. • If the student's graphs look linear due to scale issues then we will ask them to leave it as is and create another one with an appropriate scale for comparison. • At this point they will leave their equations as they are and we will lead them to correct equations later in the discussion portion of the lesson.	Student work as they produce it.

Comparing and Discussing

Looking at the table first:

1. Ideally some groups will have used fractions and others decimals. This will lead to a discussion about if they represent the same area of a ballot and which is easier to visualize- the fraction or the decimal. Example: Cut 10 is 1/16 as well as .0625. Another discussion point will be which form helps them recognize the pattern of decay.
2. This will lead to a discussion of the pattern in general.
3. Discuss where the table begins (0 cuts 64 in.^{2}) and why?

Looking at the graph next:

1. How does the graph relate to the data from the table?
2. What is the shape of your graph?
3. What do the dots on your graph represent?
4. Why is the spacing so far apart initially and closer together as the paper is cut more?
5. How does your graph show decay?
6. Will the area of your ballot ever get to 0?
7. If a group has two graphs, one representing incorrect scale thus making their graph look linear, we will ask them to explain how they changed their graph to show exponential decay.

Looking at the equation:

1. How did you determine the initial size of the paper?
2. How is that represented in the table? How is that represented in the graph?
3. How should it be represented in the equation?
4. How is the dependent variable related to the table? How is the dependent variable related to the graph?
5. How is the dependent variable related to the equation?

Summing Up **What was today's situation?** Possible response: We mathematically explored the relationship between the area of a ballot and the number of times a paper is cut to make that ballot. **What are the essential things we found out about this situation? We hope the students will come up with the following:** We found that this situation represented an example of what is termed exponential decay (the area of each ballot got rapidly smaller, halved, as each cut was made. This is the inverse of exponential growth situations that are the ones we have been studying in this unit. This relationship was shown on our graphs by a rapidly descending curved shape that gets close to zero then flattens out. We decided that the curve would never actually reach zero even though it would get very close. We determined that to find the next point on our graph or the next entry on our table in an exponential decay situation that we need to either divide or multiply by a fraction.	Teacher may need to give the term decay to the students for their description. How does today's situation of exponential decay compare to exponential growth? How was this relationship represented on your graph? How did we find the next point on our table or graph?	

Evaluation

Exit slips: The students will respond to the following prompts.

How are exponential growth relationships and exponential decay relationships similar? How are they different?

Group Reflection of Practice Lesson:

After the practice lesson and the debriefing many changes were considered for the public lesson. The goals of the lesson did not change. However the introduction and the posing of the problem as well as the tools made available to the students did change.

Practice Lesson:

- Group demographics and setting: All male students in their own classroom environment. There were very few visitors observing the lesson. Students were placed in small groups of four. They sat in the same groups that they had worked with for several weeks. Student generated poster of problem 1.1 with a table showing exponential growth.
- Tools made available: 8 x 8 paper ballot, scissors, rulers, calculators, large paper, and markers.
- Teacher actions: Just as in problem 1.1 the paper ballot was cut in half. As the teacher was holding the paper she instructed the students to find the area of the paper ballot. They were told they had to make cuts to determine the area of subsequent ballots.
- Student actions that led to change: Students were focused on cutting and measuring the paper ballots. In fact so much time was spent on this that not one group came close to writing an equation. They failed to see the numerical pattern in their tables that would help them to create the equation. The groups that used calculators to divide the paper ballot with an area of one and less stumbled on the decimal versus the fractional equivalents. This also impeded their progress towards the goal of writing an equation.
- Many of their struggles prompted us to reconsider what we were going to place in the supply tubs and the lesson introduction. For example we eliminated calculators. We also decided to inform the students of the size of the paper.

Public Lesson:

- Group demographics and setting: A group of all female students in the school library. Their environment was devoid of familiar aides such as anchor charts displayed on the classroom wall. There were approximately thirty visitors observing the lesson. Students were placed in small groups of four. They were sitting with the same students they had been working with for several weeks.
- Tools made available: 8 x 8 paper ballot, scissors, rulers, large paper, and markers.

- Teacher actions: Changed the introduction from the practice lesson. Teacher referenced problem 1.1. In the posing of the problem the teacher tried to focus the students on the pattern in the areas of the ballots. The students were informed the area of the ballot was sixty-four square inches. The students were also asked to describe the area of the ballots after ten cuts, one hundred cuts and for any number (*n*) of cuts.

- Student actions that led to further modifications of the lesson:

These students tended to fold the paper and examine the ballot instead of cutting. Some of them had misconceptions about cutting the ballot in half and instead folded the paper into tenths strips. They focused more on creating the table and searching for the pattern than in measuring the paper ballots. Without calculators the students were able to come up with fractional measurements, but had a lot of difficulty working with fractions. Their work on the table allowed many of them to see the pattern and eventually quit folding and cutting.

Lesson Plan Revision (to be instructed next school year)

Steps, Learning Activities *Teacher's Questions and Expected Student Reactions*	*Teacher's Support*	*Points of Evaluation*
1. Introduction From the textbook: In Problem 1.1, you read about the ballots Chen, the secretary of the Student Government Association, is making for a meeting. Recall that Chen cuts a sheet of paper in half, stacks the two pieces and cuts them in half, stacks the resulting four pieces and cuts them in half, and so on. You investigated the pattern in the number of ballots created by each cut.	Displayed student work from Problem 1.1.	
2. Posing the Problem In this problem, you will not be considering the number of ballots, but rather you will look at the pattern in the areas of the ballots. Chen begins making ballots with a paper that has an area of 128 square inches. This looks like a 16 x 8 rectangle. Your work today is to demonstrate mathematically in as many ways as possible how the area of the ballot changes after each of 10 cuts, and then describe the change in the area after 100 cuts, and finally how would the area change for any number of cuts.	The students will be given a 2 foot by 3 foot piece of centimeter graph paper.	
3. Anticipated Student Responses Table: • We predict the students will begin their solutions by creating a table. Most of the students will label the x and y column correctly. Some of the students will indicate area in fractional form. Some of the students will indicate area in decimal form. • We predict that some students will omit the row for 0 cuts with an area of 128 in.^{2}. • We predict that some students will omit the column to show their calculations. Graph: • We predict that some students will make a graph that looks linear by using the values in the y column as their intervals on the y-axis. • We predict some students will connect the	• If the students did not begin their table with 0 cuts and 128 in.^{2} we will question them through the process of why their table should begin with this. • If the students do not have three columns (# of cuts, area and calculations) they will be asked how they have made their tables so far this year. • If the students only have the decimal answer for the area they will be questioned towards the fractional equivalent. • If the student's graphs look linear due to scale issues then we will ask them to leave it as is and to create another	Student work as they produce it.

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data points with a solid line to show continuous data and some will connect the data points with a dashed line to show discrete data.
• We predict some students will place the independent and dependent variables on the wrong axes.

Equation:
• We predict they will come up with the following:
A = l x w
A = ½ l x w
A = 128 x ½
A = 128n/2
A = (128/2)(n)
A = (2n)(128)
$A = 128(1/2)^n$
Something of the form y = mx +b
A = 128(1/2)
$A = 128(1/2)^n$
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one with an appropriate scale for comparison.
• The issue of continuous or discrete data will be resolved during the summary.
• If the variables are on the wrong axes we will question them about which variable depends on the other.

• At this point they will leave their equations as they are and we will lead them to correct equations later in the discussion portion of the lesson.
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4. Comparing and Discussing
Looking at the table first:
• Ideally some groups will have used fractions and others decimals. This will lead to a discussion about if they represent the same area of a ballot and which is easier to visualize- the fraction or the decimal. Example: Cut 10 is 1/4 as well as .25. Another discussion point will be which form helps them recognize the pattern of decay.
• This will lead to a discussion of the pattern in general.
• Discuss where the table begins (0 cuts 128 $in.^2$) and why.
• Discuss how they wrote out their calculations. Discuss the connection to exponential growth.
Looking at the graph next:
• How does the graph relate to the data from the table?
• What is the shape of your graph?
• What do the dots on your graph represent?
• Why is the spacing so far apart initially and closer together as the paper is cut more?
• How does your graph show decay?
• Will the area of your ballot ever get to 0?
• If a group has two graphs, one representing incorrect scale thus making their graph look linear, we will ask them to explain how they changed their graph to show exponential decay.
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• Is your data discrete or continuous? • How can you show discrete or continuous data? Looking at the equation: • How did you determine the initial size of the paper? • How is that represented in the table? How is that represented in the graph? • How should initial size be represented in the equation? • How is the independent variable related to the table? How is the dependent variable related to the graph? • How is the dependent variable related to the equation?		
5. Summing up **What was today's situation?** Possible response: We mathematically explored the relationship between the area of a ballot and the number of times a paper is cut to make that ballot. **What are the essential things we found out about this situation? We hope the students will come up with the following.** You found that this situation represented an example of what is termed exponential decay- the area of each ballot got rapidly smaller (in this case half) as each cut was made. This is like the inverse of exponential growth situations that are the ones we have been looking at in Investigation 1. This relationship was shown on your graphs by a rapidly descending curved shape that gets close to zero then flattens out. We decided that the curve would never actually reach zero even though it would get very close. We determined that to find the next point on our graph or the next entry on our table in an exponential decay situation that we need to either divide or multiply by a fraction.	Teacher may need to give the term decay to the student's description. How does today's situation of exponential decay compare to exponential growth? How was this relationship represented on your graph? How did we find the next point on our table or graph?	

Group Reflection of Lesson Study

Our team consisted of one eighth-grade teacher that has participated in lesson study for three cycles, one eighth-grade teacher that had never participated in lesson study, one seventh-grade teacher that had never participated in lesson study and the campus math process trainer that has participated in two lesson study cycles. The team began work on the lesson at the Chicago Summer Lesson Study Institute in August. There we worked daily on the lesson with the assistance of Dr. Takahashi, Dr. Watanabe and other lesson study teams. We continued to meet and work on the lesson throughout the year.

Our school district requires that we teach from the Connected Mathematics reform curriculum, so a lesson from this program was chosen as our research lesson. Our practice lesson revealed problems with the lesson that we had not anticipated. First, students focused on measurement which was not even part of the lesson. Afterward we discussed not telling them to find the area and ended up restructuring the entire launch to the problem. Students had used calculators to do their work and so were dealing with decimals instead of fractions which made it much more difficult for them to see a pattern in their numbers. We decided to not provide calculators in the public lesson. The practice lesson provided valuable information and upon reflection we realize that in our next lesson study cycle we definitely want to have more than one practice lesson. The public lesson also revealed areas where we could improve upon the lesson (changing to a different number to avoid the difficulty with fractions). We learned that even when starting with a good mathematical problem from a good curriculum, we need to make changes to the lesson so that our students can be successful with it.

Lesson study has given us the opportunity to go outside of our campus and work with a larger group of teachers. That allowed us to see lessons from many different viewpoints and helped us learn to focus deeply on our own lessons. Sharing teaching experiences with other teachers helped us to learn more about presenting lessons to our students. We have learned from observing the public lessons presented by other teams in our lesson study cohort. We found the debriefing panel discussions extremely valuable in helping us to delve deeper into the mathematics. The expertise of Dr. Aki Takahashi and Dr. Tad Watanabe, as well as, Dr. Susana Salamanca and the New Mexico State University personnel helped us to see and understand mathematics beyond the lesson that we never could have seen without their guidance.

Going through the process of team planning with Lesson Study helped us realize that two, three or more heads are better than one. The more ideas and discussions of a lesson, the better the lesson will be. Teacher support is valuable. Collaboration enables us to better prepare our lesson. A better prepared lesson means increased learning for the students. Also, when teachers model learning together and collaboration, students learn to value the collaborative process as well. They see that working as a team is important when you are an adult and not just when you are a student. Even when we are learning together we are modeling for our students as well.

Personal Reflection:
Ana Castro
STMS 2007-2008

Being part of this lesson study group has been a wonderful experience. The opportunity to participate in the Lesson Study Summer Institute in Chicago this past August has given me insight into planning appropriate lessons, modifying existing lessons, and implementing them. The sense of teamwork that my colleagues and I have developed is based on the same interest: to learn. My colleagues have taught me, through their experience, how to implement new ideas in my classroom. The chance to work so closely with other teachers has improved my confidence and my understanding of how children learn mathematics.

When we were in Chicago discussing which lesson we would work on we felt confident about the choice we made. Then when it was brought up to the other groups we felt challenged because the high school group suggested that this lesson as written in the textbook would lead the students to misconceptions about the math. We decided to pursue the lesson anyway and have learned a great deal about mathematics and children through the process.

In the research lesson, I noticed that the kids struggled with some issues that we had not anticipated. One of the issues that I observed was that the students had a hard time finding the initial area of the ballot. I think the confusion was perpetuated by the instructions that we planned for them. Another issue was the manipulatives that we used. I have realized that the results of these issues was that the students started to measure the ballots and got more focused on finding the area of the ballots instead of figuring out the mathematical pattern. For the public lesson we corrected a few things. By changing some of the tools available to the students I observed that the students focused more on the mathematical pattern.

Being part of this lesson study group has impacted my life in many ways. Working with people like Akihiko Takahashi, Tad Watanabe, the Chicago lesson study staff, and Santa Teresa Middle School Team has been a terrific, educational, knowledge building, and challenging experience. I really believe that every person that is remotely interested in learning how to improve his or her teaching practice should experience this process. This will help them to promote the thinking, processing and problem solving skills of each student they come in contact with.

Personal Reflection
Corina Medrano
STMS 2007-2008

Working on the lesson on exponential decrease at first seemed to leave us with doubt as to whether we should continue with our lesson. People at the conference in Chicago felt that introducing exponential decrease was not as easy as we anticipated and that we might be giving the students the wrong idea of the concept. In spite of the negative feedback we proceeded with our lesson.

Our research lesson helped us to see the need for changing the public lesson. We eliminated the use of calculators and scissors. The use of these tools took up too much time and the concept of the lesson was lost to a certain point. Since so much time was taken up with the cutting and measuring of the ballot we did not get as far into the lesson as we had anticipated.

During the revision of the public lesson we came out with a totally different way of implementing the lesson. Unfortunately we did not have the time to present the revised public lesson but we will be doing it next year.

Lesson Study has helped me to better understand planning of mathematics lessons. It would be extremely beneficial to our students if we could have sufficient time to plan each lesson accordingly. The details involved in such a lesson leads to a better-prepared instructor. One who tries to foresee all the problems and anticipates ways to guide students as well as all the advantages of delivering a more effective lesson.

Personal Reflection
Constance Sherk
STMS 2007-2008

I was privileged to be part of the Santa Teresa Middle School Lesson Study Team during the 2007/08 school year. This was my second year as a member of a lesson study group, but the first year for this team. The lesson study process gave me an opportunity to look deeply into a math lesson and question the lesson and the instruction of it. The fact that I had teammates and experts to help me through this process was a definite advantage.

We started our lesson study planning in August at the Chicago Summer Lesson Study Institute. There we had the opportunity to work with Dr. Takahashi, Dr. Watanabe and other lesson study teams to help us get started on our lesson. We chose a lesson from our district curriculum, *The Connected Mathematics Program*, on exponential decay. In Chicago, we were able to get a good start on the planning of the lesson with excellent feedback from fellow lesson study teams. The questions and concerns they put forth made us think more deeply about the lesson we were preparing. The presentations by lesson study experts helped us work through the process as well. We came back to our campus with a lot of the initial work on the lesson complete.

Once back on campus, we continued to work together to plan the lesson, going over details about materials, questions, numbers, and how to pose the problem to the students. We continued to have discussions at team meetings through the month of August and into September. We had agreed to present our public lesson on September 27th, so did not have a long time to continue the planning. One member of our team did get to present a practice lesson to her class and we were able to make some changes to the lesson based on how this lesson went and the debriefing conversation that followed.

The public lesson was presented on September 27th to a class of 8th grade girls. We learned a lot about this lesson. The girls really struggled with the fraction knowledge that this lesson required. Some groups were not able to get past this struggle, but others were able to work through the problem and meet the goals of the lesson. The debriefing conversations brought out the idea of using numbers that would not require so much fraction work. This change might give us more information on whether or not students grasped the concept of exponential decay, the goal of the lesson. We plan to try this suggestion when teaching the revision of this lesson next school year.

Personal Reflection
Kim Wollard
STMS 2007-2008

Lesson study began for me in the summer of 2007. I attended the Park City Mathematics Institute (PCMI) and it was there that I got a glimpse of lesson study. Akihiko Takahashi was there and facilitated several days' discussion about mathematics education. The concept of Kyozaikenku was brought to my attention. This process of studying the raw materials to teach and instruct my students needed to become a habit of mind. I do believe that I am on the way to achieving this. PCMI led to Chicago and the Lesson Study Summer Institute. This is when lesson study commenced to take form in my mind. On the theoretical side, lesson study is about seeing the practical application of new ideas. It is about seeing ideas, revising them, and revisiting them. It is student focused and teacher led. It is an active process that involves the collaboration of team members. On the practical side all of these ideas came to be part of the process. I do not think that I have ever been so purposeful about a mathematical lesson. Everything from choosing a lesson, considering the introduction, the student's explorations, anticipated responses, and potential misconceptions was thought through.

As part of the team I wrestled with many choices regarding all of these ideas. It seems as if every decision that was made concluded with a question of why. Why did we decide to use the problem we did? Why did we allow certain materials and manipulatives? Why did we word key questions the way we did? Did the lesson tie in with our goals? Through all of this discourse, planning, action, and revision I learned a tremendous amount about myself, my students and the teaching of mathematical concepts. One of my biggest goals as an educator should be to help my students develop their mathematical thinking. Lesson study is the process to accomplish this. Prior to participating in lesson study I would observe other mathematical lessons and focus more on the teacher and what he or she were doing. I wanted to become a better teacher so I watched other teachers. I have now discovered that by watching how students interact with mathematics and each other will help me become a better teacher.

Since I will be teaching the particular lesson for many years to come I will consider all of the discussions my team and I have had about it. Lesson study will have a lasting impact on my teaching. This is a process that applies to daily life in the classroom. It has helped me become more mindful of what occurs during lessons. Long after my students have walked out of my life and moved on to bigger, brighter and tougher mathematical situations, the residue of what we have accomplished together needs to linger. What will that residue be? What will be left after everything else gone? What I hope the residue will be is that they recognize the value of struggling with mathematical concepts. That they realize there is value in questioning, challenging, and reflecting on the mathematics.
At times this year of lesson study has been mentally exhausting. I have expended so much energy debating the mathematics of my curriculum. I feel empowered to make good choices about what I do in my classroom.