Section 2: Lyle's class

Lyle reads the problem to the group and asks for students to share their solutions. Since none of the students volunteers to show his or her work, Lyle reads the problem aloud again. He asks how many people had worked on the problem. With the exception of two students, everyone raises a hand.

Lyle: So almost everyone worked on the problem, but no one is volunteering!
Naomi: I don't think my answer is what you want, like proving—like real proving.

She illustrates the same folding approach that Kevin had shown in Tom's class.

Lyle (*addressing the class*): What do you think? Is this demonstration enough to believe that any of the medians in this triangle divides the triangle into two regions of equal area?

Samantha: Are you saying sides AB and AC are equal?
Naomi (*elaborating on her argument*): Yes, I said it was okay for this median (*pointing at* AM_1), the one that cuts the unequal side, but it is not true for the other ones. Like when I drew BM_2, the two triangles did not match [*see* ***fig. 2***].
Meri: But the problem did not say that they had to be congruent; it said that their areas had to be equal. Two triangles can have different measurements but still have the same area.
Josh: But if ABC is an equilateral triangle then all the triangles are congruent.
Meri (*responding to Josh*): Yes, but this problem did not say that they HAD to be congruent! It said that they had to have the same area!

Everyone is silent for almost a minute. Lyle addresses the group again.

Lyle: These are both excellent comments. Josh says in an equilateral triangle, each median divides the triangle into two congruent triangles. Meri says that in our original statement it does not matter if the triangles are congruent—all we need to check is if their areas are equal. What do you think?
Samantha: I think they are both right. (*Several other students nod their heads in agreement*). I was talking to Noah about this. He can explain it better.
Will: I know what I did wrong. I thought they both had to be congruent, so I said the statement was wrong; but now that I listened to Meri, I think she

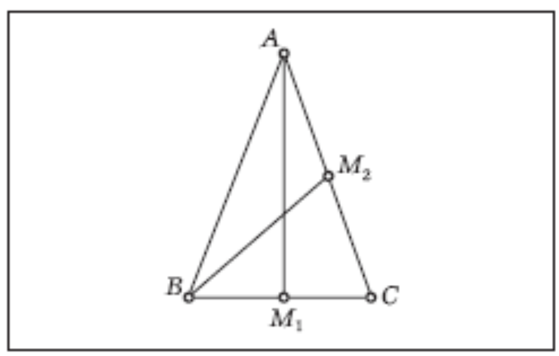

Fig. 2 Naomi's picture

The transcript is excerpted from A. Manouchehri, and D. St. John, "From Classroom Discussions to Group Discourse," *Mathematics Teacher*, NCTM, Vol 99, No. 8, 2006 pp. 546-547.

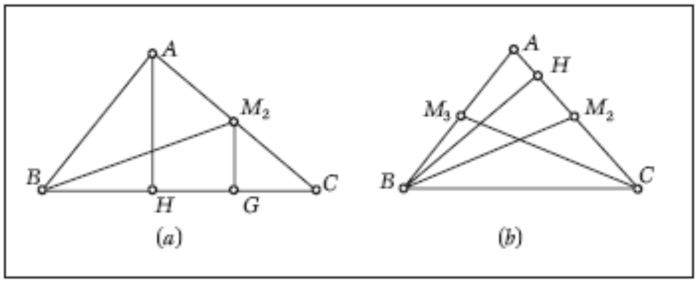

Fig. 3 Noah's model

is right. Like, we can have a triangle with height and base of 5 and 6, another one with height and base of 10 and 3. They both have the same area but they are not equal.

Noah: I think Josh is right about all 6 triangles being equal because here is what I did: I looked at the triangle and one of its medians and saw that the two triangles inside have the same altitude.

Lyle draws the altitude AH, perpendicular to side BC as illustrated in ***figure 3a****. Noah corrects Lyle.*

Noah: No, those are not the ones I looked at (*goes to the board and produces* ***fig. 3b***). Then I said the areas of ABM_2 and CBM_2 are the same. See, $AM_2 = CM_2$ and BH is their common height. Here it really does not matter which median we consider, it always gives us two triangles of equal area. See, I could have looked at this other median (*draws* CM_3). Again, the altitude is the same for these two (*pointing at triangles* CAM_3 *and* CBM_3), their bases are equal, so their areas are equal [*see* ***fig. 3b***].

Hope: So what you are saying is that the statement is true for all triangles. It does not matter what kind of triangle we have, it does not matter if it is isosceles or not—because Noah's argument did not depend on the lengths of the sides of triangle ABC. So we could use the same method to show the areas would be equal in any triangle.

Noah: I guess you are right. I did not see that, but it is right.

Lyle: Good! Let's go back and see if there is a different way of looking at this problem. I am curious to know how we might justify Naomi's method.

Megan: Didn't we show that in an isosceles triangle the median is also the altitude? I mean the median that cuts the unequal side. So, with that we know triangles ABM_1 and ACM_1 are congruent by SSS congruency theorem.

Sounds of recognition arise from the group.

Jamie: When you drew those altitudes, AH and M_2G, I saw something. I thought, okay, the ratio of the altitudes is 2 to 1, because the ratio of AC to M_2C is 2 to 1. So, since the altitude of BM_2C is a half of AH, the area of triangle BM_2C is exactly a half of the area of ABC, so that leaves only a half for the area of triangle BM_2C. So, they must be equal.

Rosha: But Jamie, we don't know the ratio of the altitudes is 1/2. They may not be 1/2.

Jamie: I think it is 1/2 because M_2 is the midpoint of side BC.

Lyle steps away from the board and appears to be thinking about Jamie's method. Several students ask if

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Lyle could give them a hint on how to prove Jamie's proposition.

Lyle: I never considered proving it this way

He draws a line from M_2 parallel to BC. Jamie and several others simultaneously shout "similar triangles," referencing their discussion of medial triangles from two weeks earlier. [Earlier in the term, students had considered the relationship between a triangle and its medial triangle, formed by connecting midpoints of its sides. At that time, the theorem stating that a segment connecting the midpoints of two sides of a triangle is parallel to the third side.]

Morgan: I think we can use this approach to show Naomi's folding method, because once we draw the medial triangle then we can show that areas of the triangles inside are equal. Like this. (*She produces* ***fig. 4*** *to illustrate her point.*)

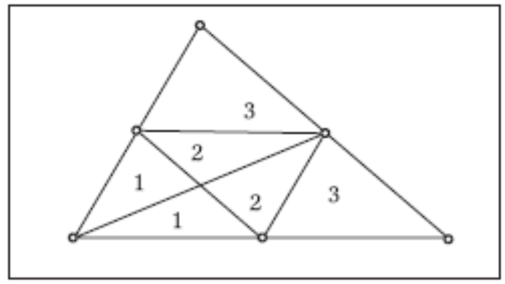

Fig. 4 Morgan's visual representation of Solmaz's conjecture

Lyle advises students to get into small groups and review the different procedures that were presented in class. He instructs them to revisit each method and to decide, first, if each of the suggested arguments was complete and then, to refine each one if needed.

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