Section 1: Tom's class

Tom reads the problem to the group and asks if students had worked on the problem. None of the students volunteers to show his or her work, so Tom encourages them to participate.

Tom: I don't want to just show you how to solve this problem. We need a volunteer. It doesn't matter if you have not solved it completely.

*One of the students, Kevin, volunteers to share his work. He instructs Tom to draw a triangle on the board and to label its vertices. Tom asks Kevin to come to the board to present his work. Kevin produces a drawing and explains it [see **fig. 1a**].*

Kevin: I said when we fold the triangle about the median AM_1, the areas of these two regions (*points at triangles* ABM_1 *and* ACM_1) match, so they are equal.

Tom looks at the group; but since no one reacts to Kevin's approach, he addresses Kevin.

Tom: What did you assume about the triangle? The problem says "isosceles triangles." Which of the sides are equal?
Kevin: Sides *AB*, *AC*, and *BC*.
Tom: So, what type of triangle is it? If all three sides are equal, what kind of triangle is it?

A student shouts out that it is equilateral.

The transcript is excerpted from A. Manouchehri, and D. St. John, "From Classroom Discussions to Group Discourse," *Mathematics Teacher*, NCTM, Vol 99, No. 8, 2006 pp. 546-547.

Tom (*addressing the group*): You need to remember that not all isosceles triangles are equilateral. The problem said "isosceles triangle."
Evan: So your method does not work? Your method only works when it is equilateral?
Kevin (*responding to Evan*): If I changed it to just sides AB and AC equal then we can still fold it and they match.

Several students nod their heads in agreement. One student holds up a piece of paper on which she had drawn an isosceles triangle folded along the median AM_1 to illustrate Kevin's reasoning.

Tom: This folding method shows the two triangles are equal only for this case (*pointing at the median* AM_1), but it doesn't work when we consider another median, say BM_2, because in this case the triangles don't match. Do you see my point? Evan, did I answer your question?
Evan: Yes, I see why he is wrong.
Solmaz: I drew all three medians and tried to see if the triangles, the six triangles inside, ended up being equal, but I did not finish.
Tom: Okay, does anyone else have a solution?
Nassim: I don't know how we can find the areas without knowing the size of the triangle.
Tom: Do we need specific measurements to verify this statement?

Nassim does not respond.

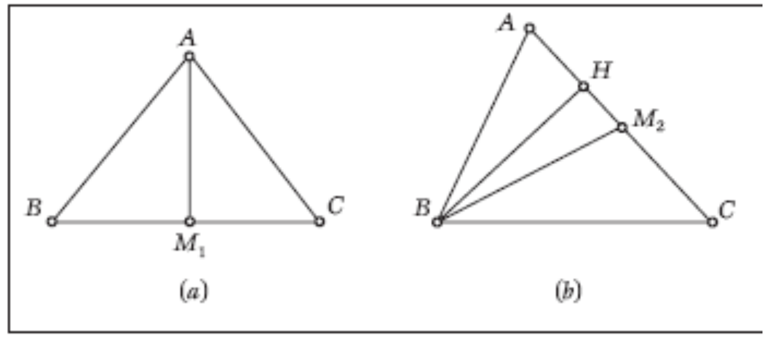

Fig. 1 Initial models used in Tom's class

Sara: I said when we draw a median we create two triangles in the original triangle. The bases of these two triangles are equal because of the property of midpoint, and then they share the same altitude, so if we use area = 1/2(height • base), then we get the same areas.

Tom draws a triangle to illustrate Sara's argument.

Tom: Sara says if we use the definition of the median and consider the common altitude of triangles ABM_2 and BCM_2, then we have equal areas [*see* ***fig. 1b***]. This is really slick: a generalized argument that applies to any of the medians in the triangle, right? Do you see it?

A student jokingly refers to Sara as the "brains" and asks how she came up with the idea. Sara explains that the hardest thing for her was noticing that triangles ABM_2 and BCM_2 shared the same altitude. Tom asks students to finish recording Sara's proof. Students are then advised to move on to the next homework problem.

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