Ceci n'est pas un headnote.

1 *Sum-mer Vacation*

Hey, welcome to the class. We know you'll learn a lot of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How did others at your table think about this question?
- **Respect everyone's views.** Remember that you have something to learn from everyone else. Remember that everyone works at a different pace.
- **Learn from others.** Give everyone the chance to discover, and look to your tablemates for new perspectives on problems. Resist the temptation to tell others the answers if they aren't ready to hear them yet. If you think it's a good time to teach your tablemates about Dirichlet characters, think again: the problems should lead to the appropriate mathematics rather than requiring it. The same goes for technology: the problems should lead to the appropriate use of technology rather than requiring it.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff, and maybe other stuff sometimes. Check out Important Stuff first. All the mathematics that is central to the course can be found and developed in the Important Stuff. *That's* why it's Important Stuff. Everything else is just neat or tough. If you didn't get through the Important Stuff, we probably noticed... and that question will be seen again soon. Each problem set is based on what happened before, either in problems or in class discussions.

At least one problem in an upcoming problem set is unsolved. You should solve it!

On Day 3, go back and read these again.

Will you remember? Maybe!

Important Stuff

Every positive integer has divisors, numbers that divide evenly into it. The divisors of 4 are 1, 2, and 4. The divisors of 18 are 1, 2, 3, 9, and 18, and maybe one more.

Make sure you bring divisors if you're going to da beach and da sun is out.

We're live, investigating a function called σ, which takes in a positive integer, and spits back the *sum* of all its divisors. For example, $\sigma(4) = 7$ and $\sigma(18) \geq 33$.

PROBLEM

Stuff in boxes is more important than other Important Stuff! By the way, the pencil was invented around 1560.

Here's a massive table for the σ function. Complete the table without using any technology developed after 1565.

n	1	2	3	4	5	6	7	8	9	10	11	12
$\sigma(n)$				7								
n	13	14	15	16	17	18	19	20	21	22	23	24
$\sigma(n)$												
n	25	26	27	28	29	30	31	32	33	34	35	36
$\sigma(n)$												
n	37	38	39	40	41	42	43	44	45	46	47	48
$\sigma(n)$												
n	49	50	51	52	53	54	55	56	57	58	59	60
$\sigma(n)$					54							
n	61	62	63	64	65	66	67	68	69	70	71	72
$\sigma(n)$												
n	73	74	75	76	77	78	79	80	81	82	83	84
$\sigma(n)$												
n	85	86	87	88	89	90	91	92	93	94	95	96
$\sigma(n)$												

1. Describe some patterns in the table for the σ function, especially patterns that helped you complete the table quickly, or patterns you could use to find other outputs.

2. Determine each of the following. Hey, stop trying to use a calculator!
(a) $\sigma(128)$
(b) $\sigma(243)$
(c) $\sigma(5 \cdot 49)$
(d) $\sigma(257)$
(e) $\sigma(1001)$

5 times 49? You're making it way more complic. . . Ohhhhh. Cool.

3. Define $A(n) = \frac{\sigma(n)}{n}$. Alright, fine, you can start using a calculator now.
(a) Find all numbers n with $A(n) \leq 1$.
(b) Find three numbers n with $A(n) = 2$.
(c) Are there any numbers n with $A(n) = 3$?

Neat Stuff

Here are some more good questions to think about.

4. If p is prime, what can you say about $A(p)$?

I heard $A(p)$ is a freeloader.

5. If p and q are primes, what can you say about $A(pq)$?

6. If p and q are primes, find the maximum possible value of $A(pq)$.

7. Without relying on technological crutches, find a number for which $\sigma(n) = 1000$ or show that no such number exists.

8. Find the maximum possible value of $A(n)$.

9. Go back to the problems on page 2, except now use $\sigma_2(n)$, the sum of the *squares* of the divisors, and $B(n) = \frac{\sigma_2(n)}{n^2}$.

Tough Stuff

Here are four much more difficult problems to try.

10. Find all numbers for which $\sigma(n) = 1000$.

11. Find a number n for which $A(n) \geq 10$, or prove that no such number exists.

12. Find an odd number for which $A(n) = 2$, or prove that no such number exists.

13. Find the maximum possible value of $B(n)$.

It's on the number line somewhere, so how hard can it be to find, right? Right?

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