4 *Reappearing Doodads*

PROBLEM

We're going to start with doing the same thing, over and over. Find a closed rule for $T(n)$. If you're not sure where to begin, consider taking common ratios of consecutive terms.

$$T(n) = \begin{cases} 2 & \text{if } n = 0 \\ 13 & \text{if } n = 1 \\ 13T(n-1) - 30T(n-2) & \text{if } n > 1 \end{cases}$$

Lately the most common reappearing doodad has been a vuvuzBRAAAAA-WWWWWWWWWWWWWW

Important Stuff

1. Calculate each of these.

(a) $\left(\frac{1+\sqrt{5}}{2}\right) + \left(\frac{1-\sqrt{5}}{2}\right)$

(b) $\left(\frac{1+\sqrt{5}}{2}\right) \cdot \left(\frac{1-\sqrt{5}}{2}\right)$

Sheesh, fractions and radicals? It's only problem 1!

2. Find two numbers with sum *salt* and product *pepa*.
(j) $salt = 13, pepa = 30$
(a) $salt = 10, pepa = 21$
(n) $salt = 100, pepa = -1469$
(e) $salt = 1, pepa = -1$

Whatta number, whatta number, whatta mighty good number!

3. Function $S(n)$ is defined by this recursive rule.

$$S(n) = \begin{cases} 1 & \text{if } n = 0 \\ k & \text{if } n = 1 \\ 10S(n-1) - 21S(n-2) & \text{if } n > 1 \end{cases}$$

Find all possible values of k for which this is a rootin-tootin geometric sequence.

If you get this one right, you're golden. Problems about Spinderella may appear later.

4. Find the solution to each of these systems of equations.
(s) $A(1,7) + B(1,3) = (2,10)$
(e) $A(1,7) + B(1,3) = (1,19)$
(t) $A(1,7) + B(1,3) = (0,1)$
(h) $A(1,7) + B(1,3) = (h,k)$

Forgot what that stuff means? We defined this shorthand previously: $(1,5) + (1,2) = (2,7)$ and $2(2,7) = (4,14)$.

5. Function $T(n)$ is defined by this recursive rule.

$$T(n) = \begin{cases} h & \text{if } n = 0 \\ k & \text{if } n = 1 \\ 10T(n-1) - 21T(n-2) & \text{if } n > 1 \end{cases}$$

Find a closed rule for $T(n)$ if...

(a) $h = 2, k = 10$
(b) $h = 1, k = 19$
(c) $h = 0, k = 1$
(d) you're not given h or k

The last answer will be in terms of h and k, which are grateful to finally be used for something other than horizontal and vertical shifts.

6. Recursion can happen to points, too! The recursive rule

$$(x, y) \mapsto (y, x + y)$$

takes a point and produces a new point. This recursion gives a whole sequence of points, as long as you have a place to start. Let $(0, 1)$ be the starting point.

(k) Show that the next point in the sequence is $(1, 1)$.
(i) Show that the *next* point in the sequence is $(1, 2)$.
(m) Determine the next eight points in the sequence.

So for example, $(10, 19)$ goes to $(19, 29)$ goes to $(29, 48)$ goes to ...

Just let it. $(0, 1)$ deserves nice things too. In case you were wondering, there are two Kims.

7. It's *Technology Time* with our very special host, the TI-Nspire! Yay! In today's Technology Time you will make a spreadsheet with the points from problem 6.

- Turn the calculator on by hitting the HOME icon, then hit it again to call up the main menu.
- Select the spreadsheet icon. You should now have a blank spreadsheet.
- In cell A1 (note: not the top row!) type **0**. In cell B1 type **1**.
- The rule is $(x, y) \mapsto (y, x + y)$, so use the formulas to type the next row: in cell A2 type **=b1**. (The equals key is in the upper left.) This carries the value from cell B1 into cell A2. The number 1 should appear.
- In cell B2 type **=a1+b1**. This carries the sum of cells A1 and B1 into cell B2. The number 1 should appear.
- Now for the cool part: FILL DOWN! Go to cell A2 and hold down the "clicker" (big button in the middle) for about 2 seconds. Cell A2 should now have a dotted line around it. Now click down a ways (to row 10). You're filling down the formula! Hit enter to confirm.
- ...but the formula from the second column needs to come down, too. Fill it down as well. Woot!

It's fun to play with spreadsheets. It's fun to play with spreadsheets. It's fun to play with spreadsheets, and this is how you do it!!

To select something, use the button in the center of the pad, or hit the ENTER key on the right.

It's fun to play with spreadsheets. It's fun to play with spreadsheets. It's fun to play with spreadsheets, and that's how you do it!! (FTW?)

Neat Stuff

8. Consider the function $J(n)$ from yesterday:

$$J(n) = \begin{cases} 2 & \text{if } n = 0 \\ 7 & \text{if } n = 1 \\ 7J(n-1) - 10J(n-2) & \text{if } n > 1 \end{cases}$$

You saw that as n increases, the *common ratio* of consecutive terms approached 5. But what happens if n decreases?

(a) Determine the value of $J(-1)$ that would allow the recursive rule to continue working. Specifically, $J(1) = 7J(0) - 10J(-1)$ gives this value.

(b) Determine $J(-2)$ through $J(-6)$ to a reasonable number of decimal places.

(c) What happens to the ratio of consecutive terms as n becomes more and more negative? The ratio is always more than 1, by the way.

Remember, the common ratio would be $J(-5)/J(-6)$, not the other way around.

9. The golden ratio ϕ is the number $\frac{1+\sqrt{5}}{2}$.

(z) Show that $\phi^2 = \phi + 1$.

(a) Show that $\phi^3 = \text{blah}\phi + \text{bleh}$. Oops, we forgot the numbers! You figure it out.

(c) Find cool rules for ϕ^n for increasing values of n. How awesome is that?

(k) Find cool rules for ϕ^n for *negative* values of n.

10. Something interesting happens when you take the product of two Fibonacci numbers that surround a third. What is it?

It's it. What is it? It's . . . a song by Faith No More. . . or a delicious ice cream sandwich!

11. Find a closed rule for $R(n)$.

$$R(n) = \begin{cases} 2 & \text{if } n = 0 \\ 10 & \text{if } n = 1 \\ 10R(n-1) - 22R(n-2) & \text{if } n > 1 \end{cases}$$

12. Find a closed rule for $I(n)$.

Deeper into the problem sets, things always seem to get more complex.

$$I(n) = \begin{cases} 2 & \text{if } n = 0 \\ 10 & \text{if } n = 1 \\ 10I(n-1) - 29I(n-2) & \text{if } n > 1 \end{cases}$$

13. Find a closed rule for the Lucas numbers.

14. What happens to the sequence of units digits of Fibonacci numbers? The sequence begins 0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0…

How many Fibonacci numbers does it take to screw in a BRAAAAAW-WWWWWWWWWWWWW

15. (a) Find a closed rule for $Y(n)$.

$$Y(n) = \begin{cases} 1 & \text{if } n = 0 \\ 3 & \text{if } n = 1 \\ 6Y(n-1) - 9Y(n-2) & \text{if } n > 1 \end{cases}$$

(b) Find a closed rule for $Z(n)$.

Darn, joke interrupted by vuvuBRAAAAAWWWWW-WWWWWWWW

$$Z(n) = \begin{cases} 1 & \text{if } n = 0 \\ 6 & \text{if } n = 1 \\ 6Z(n-1) - 9Z(n-2) & \text{if } n > 1 \end{cases}$$

(c) What's going on? Research more starting pairs.

I say: hey-ey-ey-ey-ey, hey-ey-ey. I say hey! What's going on? And then I wake in the morning and I step outside…

16. Algebraically prove each of these identities. What might they be useful for, pray tell?

(a) $x^n + y^n = (x+y)\left(x^{n-1} + y^{n-1}\right) - xy\left(x^{n-2} + y^{n-2}\right)$

(b) $Ax^n + By^n = (x+y)\left(Ax^{n-1} + By^{n-1}\right) - xy\left(Ax^{n-2} + By^{n-2}\right)$

Tough Stuff

17. The *Onemorenacci sequence* is defined by the rule

$$O(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ O(n-1) + O(n-2) + 1 & \text{if } n > 1 \end{cases}$$

Find a closed rule for the Onemorenacci sequence.

18. Prove that any positive integer can be written *in exactly one way* as the sum of one or more non-consecutive Fibonacci numbers. For example: $53 = 34 + 13 + 5 + 1$.

20-22. Do problems 22 through 24 from yesterday.