6 *So You Think You Can Count*

Important Stuff

PROBLEM

Jesse buys her first car, a Mini Cooper valued at \$20,000. To pay for the car, she takes out a loan for 48 equal monthly payments at 6% APR. The loan starts at \$20,000, and each month the remaining loan balance grows by 0.5%, then the payment is taken.

1. Explain why Jesse's monthly payment must be more than \$400.
2. Find Jesse's payment to the nearest penny. Use technology but no formulas.

At our last PD, somebody set up a "car payment calculator". Don't do that. You're going to build this formula for us. Darryl thinks this formula is the bomb. Bowen thinks it's off the chain.

1. Sam buys his first car, a Nissan Versa stick-shift valued at \$10,000. Sam's loan is similar to Jesse's: 48 months at 6% APR. Determine Sam's monthly payment to the nearest penny. Notice anything?

2. The recursive rule

$$(x, y) \mapsto (y, -21x + 10y)$$

takes a point and produces a new point. This recursion gives a whole sequence of points, as long as you have a place to start. For each of these starting points, find the next three points in the sequence.

Do these in "down" order: first $(2, 10)$ then $(4, 20)$ and so on. Beware, using the point $(4, 20)$ may result in classroom snickering.

(n) $(2, 10)$	(o) $(1, 3)$
(i) $(4, 20)$	(l) $(a, 3a)$
(c) $(10/21, 2)$	(e) $(1, 7)$

3. In the last two sessions you worked on this recursion for points:

$$(x, y) \mapsto (y, x + y)$$

(a) Find a *fixed point* for the recursion, a point (a, b) that remains at (a, b) after the transformation.

(b) Find a different point that is a *scaled point* for the recursion, a point (a, b) that is taken to (ka, kb) for some number k.

Fixed points are also scaled points. Why?

4. Here's a handy notation for general stuff like the stuff in the first two problems above.

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 3y \\ 4x + 5y \end{bmatrix}$$

Evaluate each of the following using the notation above. Check your work with a secret calculating device.

(m) $\begin{bmatrix} 0 & 1 \\ -21 & 10 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 \\ -21 & 10 \end{bmatrix} \begin{bmatrix} 10 \\ 58 \end{bmatrix}$

(l) $\begin{bmatrix} 0 & 1 \\ -21 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(n) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

(i) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 34 \\ 55 \end{bmatrix}$

(ε) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 34 \\ 8 & 55 \end{bmatrix}$

Hey, this last one's different. What happen ?

5. For this function, determine the sum of $\varepsilon(0)$ through $\varepsilon(8)$.

$$\varepsilon(n) = \begin{cases} 1, & n = 0 \\ 3\varepsilon(n-1), & n > 0 \end{cases}$$

6. It's *Technology Time* again! Get your TI-Nspire main screen turn on. We're going to build the function from problem 5, other than that fancy ε thingy.

That's TI-*Nspire*, not *Inspire*. It's patented!

- From the main menu, select a Calculator screen.
- Push **menu** and select "Define" from the "Actions" menu. The word **Define** should appear on screen.
- Type **e(n) =** . Do not hit enter.

- Push the button in the upper right that looks like an absolute value symbol plus some other curly thing. A set of templates should appear on screen. Ooh, it's MathType!
- ...except it works! Select the "piecewise function" template, which is the 7th across in the first row. Hit enter and *kapow!* a template with four boxes should appear on screen.
- Make the line look like this:

$$e(n) = \begin{cases} 1, & n = 0 \\ 3e(n-1), & n > 0 \end{cases}$$

- Hit enter. The response should be **Done**.
- Type **e(8)** and hit **enter**. Woo! Compute the sum of $e(0)$ through $e(8)$ and you're done.

If you have a better description for this, let us know.

Move between all your boxes using the **tab** key on the left. For any greater than or less than symbol, the base is the "equal" key. Hit the **ctrl** key then the "equal" key. These pop-up menus often group items where they belong. To us, this is much simpler than the old Nspire layout.

Neat Stuff

7. Ashli is investigating other possible cars and payments. If she pays \$400 per month at 6% APR interest on a car that costs \$P, her remaining balance after n months can be modeled by the function

$$B(n) = \begin{cases} P, & n = 0 \\ blah \cdot B(n-1) - 400, & n > 0 \end{cases}$$

(a) Dangit. What number is *blah*?
(b) What is Ashli's balance after 48 months if the car costs \$60,000?
(c) ...if the car costs \$70,000?
(d) ...if the car costs \$80,000?
(e) ...if the car costs \$90,000?

8. (a) Draw a triangle with points $Z(1,2), I(3,2), G(3,8)$. Determine the area of the triangle.
(b) Move *ZIG* according to the rule

$$(x, y) \mapsto (y, -10x + 7y)$$

Find the coordinates of the three new points.
(c) Draw the new triangle and compute its area.
(d) Compute the following.

$$\begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 2 & 2 & 8 \end{bmatrix}$$

Use the earlier work to help! Darryl says you have no chance to survive otherwise.

9. Find a closed rule that agrees with this recursive rule.

$$f(n) = \begin{cases} 1, & n = 0 \\ 2f(n-1) + 1, & n > 0 \end{cases}$$

10. Find a closed rule that agrees with this recursive rule.

$$f(n) = \begin{cases} 1, & n = 0 \\ 2f(n-1) + 3, & n > 0 \end{cases}$$

11. Find a closed rule that agrees with this recursive rule.

$$f(n) = \begin{cases} 1, & n = 0 \\ 2f(n-1) + K, & n > 0 \end{cases}$$

12. Find a closed rule that agrees with this recursive rule.

$$f(n) = \begin{cases} P, & n = 0 \\ 2f(n-1) + K, & n > 0 \end{cases}$$

13. Build some other polygons and transform them according to the rule in problem 8. What happens to the shape of the polygons? What happens to the area of the polygons?

14. The *Twomorenacci sequence* is defined by the rule

Is that a twomore? *It's not a twomore!!* (Best said in an Austrian accent.)

$$T(n) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ T(n-1) + T(n-2) + 2, & n > 1 \end{cases}$$

Find a closed rule for the Twomorenacci sequence.

Tough Stuff

15. Marla still claims that the numbers on either side of a Fibonacci number (from 13 above) can never be prime. Is this true? If so, prove it. Factorizations may be helpful, but not necessarily prime factorizations...

16. Rina one-ups Marla by claiming that the numbers on either side of the *square* of a Fibonacci number (from 3 above) can never be prime. What you say!

Make your time.

17. Mary n-ups Rina by claiming that the numbers on either side of *any power* of a Fibonacci number (from 3 above) can never be prime. For great justice, decide whether or not this is true.