8 *Todd and Kenny Plus Table 8*

Important Stuff

PROBLEM

Darryl buys a limited-edition version of Pixeljunk Monsters for $1,000. He uses a credit card, which charges him 12% APR interest (1% per month).

(d) If Darryl doesn't pay anything each month, how much will he owe at the end of 12 months, assuming no one has charged him additional fees or broken any pinkies?

(o) If Darryl only pays $1 each month, how much *less* will he owe at the end of 12 months than if he paid nothing at all?

(n) If Darryl pays $2 each month, how much less will he owe at the end of 12 months than if he paid nothing at all?

Apparently, Darryl loves Monsters more than life itself! Well, pretty close anyway.

1. Use slopes and intercepts and whatnot to determine the monthly payment Darryl should make to pay off his Monsters game in the 12 months.

2. Let $f(n) = 1.01n - p$.

(c) Compute $f(1000)$.

(a) Write a rule for $f(f(1000))$. Collect terms but *do not evaluate anything*. If you see 1.01^2, and you will, leave it that way.

(r) Write a rule for $f(f(f(1000)))$. Again, collect terms without evaluating.

(o) Keep doing this until you get into a good rhythm, then write a rule for $f^{12}(1000)$, which means taking f and repeating it 12 times. Collect terms and try to simplify, but do not evaluate.

(l) Find the value of p that makes $f^{12}(1000) = 0$.

It's okay if an answer has some p in it.

If you can't be bothered to write all these 1.01s, maybe write k instead. While you're at it, give the 1000 a name. Besides "Bob."

3. Ben loves TaB. Each can of TaB has 46.8 mg of caffeine. Starting caffeine-free, Ben drinks a can of TaB every hour. Suppose the body removes 10% of the caffeine in the bloodstream each hour (close to reality).

(a) Write a recursive rule involving the number 0.9 for the amount of caffeine in Ben's body after n hours after his first TaB.

(b) Determine the amount of caffeine in Ben's body at 6 hours, 12 hours, 24 hours, and 100 hours.

4. Repeat problem 3, except this time Ben is coming off a caffeine bender and has 800 mg of caffeine in his system.

5. Solve the equation $B = 0.9B + 46.8$. What is special about this exact amount of caffeine?

6. Here's a recursive rule for $W(n)$:

$$W(n) = \begin{cases} A, & n = 0 \\ 0.5W(n-1) + 3, & n > 0 \end{cases}$$

The behavior of $W(n)$ depends on the value of A.

(a) Solve the equation $W = 0.5W + 3$.

(b) Find $W(0)$ through $W(5)$ given $A = 14$, and plot these values on a number line.

(c) Find $W(0)$ through $W(5)$ given $A = -2$, and plot these values on a number line. Notice anything?

(d) Repeat for A=6. Why is this value called a *fixed point*?

7. More about W.

(a) Start with $A = 106$. How far away is $W(0)$ from the fixed point? How far away is $W(1)$? $W(2)$? Hmmm...

(b) Repeat for $A = -94$.

(c) Repeat for $A = 1006$.

8. Here's another recursive rule, similar to $W(n)$.

$$D(n) = \begin{cases} A, & n = 0 \\ 2D(n-1) - 6, & n > 0 \end{cases}$$

(z) Solve the equation $D = 2D - 6$.

(a) Find $D(0)$ through $D(3)$ given $A = 14$, $A = -2$, and $A = 6$.

(c) Start with $A = 106$. How far away is $D(0)$ from the fixed point? How far away is $D(1)$? $D(2)$? Interesting.

(h) Repeat for $A = -94$ and for $A = 1006$.

Mmmm, TaB. It's the uncola! Or maybe it's refreshingly crisp? Or maybe it's got sass? Really though, TaB's never been the same since they got rid of that cyclamate.

Needless to say, Ben also wakes up in the middle of the night to drink TaB on the hour.

Perhaps from too much TaB Energy Drink, now available in stores... "bitter yet quenching!" Make sure you spell it right, that's a capital B!

W starred Josh Brolin, and is the name of a hotel chain. It's twice the letter U is, except in France, where it's twice the letter V is. W also begins the popular phrase "Wah wahhhhh..."

9. Write a closed rule that matches this recursive rule.

$$Z(n) = \begin{cases} 8, & n = 0 \\ 0.5Z(n-1), & n > 0 \end{cases}$$

10. Let $X(n) = W(n) - 6$ where $W(n)$ is the function from problem 6. We picked 6 because it's the fixed point of $W(n)$.

(a) If $W(n) = 0.5W(n-1) + 3$, what is $X(n)$ in terms of $X(n-1)$? Oh, that's awfully convenient.

(b) Find a closed rule for $X(n)$ if $X(0) = 8$.

(c) Find a closed rule for $W(n)$ if $W(0) = 14$.

Neat Stuff

11. Find all fixed points for the following recurrences, or show that there aren't any.

If there's no fixed point, there's no 1 to blame.

(a) $a(n) = 2a(n-1) - 16$

(b) $b(n) = 0.9b(n-1) - 16$

(d) $d(n) = d(n-1) + 3$

12. Consider the recurrence

$$c(n) = A \cdot c(n-1) + B$$

(a) Find the fixed point of $c(n)$ in terms of A and B. When will it exist, and when won't it?

(b) Once a number is picked for $c(0)$ and run through the recurrence repeatedly, a lot of things can happen. What happens and when? Give examples—the ones from today's Important Stuff should help.

13. Find a closed rule that agrees with this recursive rule.

$$f(n) = \begin{cases} A, & n = 0 \\ kf(n-1) + B, & n > 0 \end{cases}$$

14. Find a rule that agrees with this recursive rule!!!

Make a table of values!!! You'll see it!!!

$$f(n) = \begin{cases} 1, & n = 0 \\ nf(n-1), & n > 0 \end{cases}$$

15. For each recursion on points, find all the *fixed points*, points where (x, y) maps to itself, and all the *scaled points*, points where (x, y) maps to a multiple of itself (kx, ky).

(h) $(x, y) \mapsto (-3x, -3y)$
(a) $(x, y) \mapsto (y, -x)$
(n) $(x, y) \mapsto (y, -12x + 7y)$
(f) $(x, y) \mapsto (y, 4x + 3y)$
(i) $(x, y) \mapsto (1.01x - py, y)$

16. Find all fixed points for the recursive rule

$$F(n) = F(n-1) + F(n-2) \quad \text{if } n > 1$$

17. (t) Ellie challenges you to find a recursive rule for $P(n)$ that fits the sequence $1007, 10017, 100117, 1001117, \ldots$

(i) Explain why $P(n)$ will always be a multiple of 53, regardless of n.

(m) Find a closed rule for $P(n)$.

Stupid Stuff

18. Compute the following:

This is sometimes called the $(GA)^2$ conjecture.

$$(RAH)^2 + (AH)^3 + RO(MA + (MA)^2) + (GA)^2 + OOH(LA)^2$$

and come up with additional examples.

Tough Stuff

19. Saturday was 7/3/10, interesting because it's written as $A/B/C$ with $A + B = C$. How many more times in this century will such a date happen?

The first one was in 2002. So what, you say? So what indeed.

20. Find a rule that agrees with this recursive rule.

No, the rule isn't $T(n) = .5(n + 1)! + n$. Sorry Dave.

$$T(n) = \begin{cases} 0, & n = 0 \\ nT(n-1) + n, & n > 1 \end{cases}$$

21. Find all real numbers satisfying this system of equations:

$$a + b = cd$$

$$c + d = ab$$