10 *Real Numbers of Park City*

Important Stuff

First, the most Important Stuff of all: this Sunday, Slurpees are free. *FREE* I tells ya.

We know, it's hard to believe. But it's true! Walk in and grab one on Sunday. (A *Slurpee* is a frosty beverage offered by 7-11 in their stores. Except in Oklahoma.)

PROBLEM

In a root-beer-induced stupor, Bowen and Marla knocked over a lamp whose base was ancient Tang Dynasty pottery and whose shade was made from the wings of extinct butterflies and the pelts of 40 newborn capuchin monkeys.

They took out a \$500,000 loan to pay back Randall, Clint and Chance. They got a good fixed 4.8% APR loan and want to pay the whole thing off with 30 years of monthly payments.

Use what you've learned so far to calculate their monthly payment.

The Tang Dynasty is generally best known for feeding orange juice to astronauts.

1. Anna looks for a closed rule to match this rule for $A(n)$:

$$A(n) = \begin{cases} 104, & n = 0 \\ \frac{3}{4}A(n-1) + 10, & n > 0 \end{cases}$$

(b) Find the fixed point for the recurrence $A(n) = \frac{3}{4}A(n-1) + 10$.
(i) How far is $A(0)$ from the fixed point?
(l) How far is $A(1)$ from the fixed point?
(l) How far is $A(2)$ from the fixed point?
(t) How far is $A(3)$ from the fixed point?
(h) Describe the pattern in the results above.
(i) How far will $A(n)$ be from the fixed point?
(l) Show that $A(n) - 40 = \frac{3}{4}(A(n-1) - 40)$.
(l) Write a closed rule for $A(n)$.

So polite! Not a fraction yet. But wait: what's $\frac{27}{36}$?

2. Anna looks for a closed rule to match this rule for $C(n)$.

No, the other Anna. Coke Zero's in the hizouse again.

$$C(n) = \begin{cases} 0, & n = 0 \\ \frac{9}{10}C(n-1) + 48, & n > 0 \end{cases}$$

(j) Find the fixed point for the recurrence $C(n) = \frac{9}{10}C(n-1) + 48$.
(o) How far is $C(0)$ from the fixed point?

(y) How far is $C(1)$ from the fixed point?
(c) How far is $C(2)$ from the fixed point?
(e) How far is $C(3)$ from the fixed point?
(r) Describe the pattern in the results above.
(h) How far will $C(n)$ be from the fixed point?
(e) Show that $C(n) - 480 = \frac{9}{10}\left(C(n-1) - 480\right)$.
(e) Write a closed rule for $C(n)$.

Now *this* is recursion.

3. Here's a recursive rule for $J(n)$. Hooray for car payments.

Oh, I wish that I had Jesse's car. . .

$$J(n) = \begin{cases} 20000, & n = 0 \\ 1.005J(n-1) - p, & n > 0 \end{cases}$$

(j) Find the fixed point for the recurrence $J(n) = 1.005J(n-1) - p$.
(a) How far is $J(0)$ from the fixed point?
(m) How far is $J(1)$ from the fixed point?
(i) How far is $J(2)$ from the fixed point?
(e) How far is $J(3)$ from the fixed point?
(s) Describe the pattern in the results above.
(m) How far will $J(n)$ be from the fixed point?
(i) Show that $J(n) - \frac{p}{.005} = 1.005\left(J(n-1) - \frac{p}{.005}\right)$.
(t) Write a closed rule for $J(n)$.
(h) Determine the unique value of p for which $J(48) = 0$.

Your answers in this problem will be full of p. We are neither mature nor original.

4. Write a reclusive rule for $H(n)$ that fits

$$H(n) = 10 + \left(\frac{1}{5}\right)^n$$

Reclusive rules can be hard to find.

5. Find the fixed point for this recurrence in terms of p over r:

$$f(n) = (1 + r) \cdot f(n-1) - p$$

The Week In Review

6. Take a few minutes to look back at what you've done. List five things you learned this week, and two things you are still unsure about or would like to investigate further. We'll talk this over at the end of class.

Darryl learned how to write important shorthands like $(po^2)^2$.

Neat Stuff

7. Earlier this week, we introduced a shorthand for the recursion on points

$$(x, y) \mapsto (2x + 3y, 4x - 5y)$$

The shorthand is *the matrix*:

The Matrix is everywhere, it is all around us, even now in this very room.

$$\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 3y \\ 4x - 5y \end{bmatrix}$$

Evaluate each of these by hand.

(j) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(o) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \end{bmatrix}$ (p) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 17 \\ 42 \end{bmatrix}$

(s) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 17 \\ 42 \end{bmatrix}$

8. Use a calculator to evaluate this matrix multiplication:

Seriously, man, the Slurpee is free. We mean it!

$$\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 5 & 10 & 1 & 0 & 17 \\ 2 & 4 & 0 & 1 & 42 \end{bmatrix} = ???$$

Describe, in your own words, what the heck just happened.

Don't swallow the blue pill, okay?

9. Consider the two-step recursive rule

$$U(n) = 8U(n-1) - 16U(n-2), \qquad n > 1$$

(m) Find a closed rule for $U(n)$ if $U(0) = 1$ and $U(1) = 4$.
(a) Find a closed rule for $U(n)$ if $U(0) = 0$ and $U(1) = 4$.
(r) Find a closed rule for $U(n)$ if $U(0) = 1$ and $U(1) = 8$.
(k) Find a closed rule for $U(n)$ if $U(0) = 1$ and $U(1) = 0$.

10. Without typing data into a website box, find a closed rule for the two-term recurrence

First find the fixed point, then shift. It's like the electric slide, except way more fun, and less likely to destroy lamps.

$$H(n) = 13H(n-1) - 30H(n-2) - 360$$

where $H(0) = 0$ and $H(1) = 5$.

11. Consider the two-step recursive rule

This problem is especially easy! All you have to do is consider something.

$$v(n) = 6v(n-1) - 13v(n-2), \qquad n > 1$$

(b) Find a closed rule for $v(n)$ if $v(0) = 2$ and $v(1) = 6$.
(r) Find a closed rule for $v(n)$ if $v(0) = 0$ and $v(1) = 4$.
(y) Find a closed rule for $v(n)$ if $v(0) = 0$ and $v(1) = 1$.
(n) Find a closed rule for $v(n)$ if $v(0) = 1$ and $v(1) = 0$.
(j) Find a closed rule for $v(n)$ if $v(0) = 17$ and $v(1) = 42$.
(a) Find all scaled points for $(x, y) \mapsto (y, -13x + 6y)$.

Darn. Figures there'd be more to it than considering.

12. (a) Find a recursion in the form $(x, y) \mapsto (Ax+By, Cx+Dy)$ with *period 4*. That is, if you repeat the operation four times, you always get (x, y) back, and not before.
(b) Find a recursion in the form $(x, y) \mapsto (Ax+By, Cx+Dy)$ with period 6.
(c) Find a recursion in the form $(x, y) \mapsto (Ax+By, Cx+Dy)$ with period 8.

13. Here's a fun fun fun nonlinear recurrence.

Make sure your calculator is set to Radian Mode. Or else!

$$p(n) = \begin{cases} A, & n = 0 \\ p(n-1) - \tan(p(n-1)), & n > 0 \end{cases}$$

(w) What is $p(5)$ if $A = 1$?
(e) What is $p(5)$ if $A = 2$?
(n) What is $p(5)$ if $A = 1.96$?
(d) What is $p(5)$ if $A = 1.9$?
(y) What what!!

Can I get a...

Tough Stuff

14. Find a recursion in the form $(x, y) \mapsto (Ax + By, Cx + Dy)$ with period 5. Exact values of A through D, kthxbye.

15. Let $T_0(x) = 1$ and $T_1(x) = x$. Define the recursive rule

$$T_n(x) = 2x \cdot T_{n-1}(x) - T_{n-2}(x) \qquad n > 1$$

Yay! A sequence of polynomials! Generate a bunch and look for patterns. Try graphing them on the interval $-1 \leq x \leq 1$. Neat!

16. Suppose $f(x)$ is a cubic polynomial. Debra claims that the recursive rule

$$R(n) = f(R(n-1))$$

must have at least one fixed point. O RLY?

I can has intersection?