12 $(PA)^2$ Don't Preach

Important Stuff

PROBLEM

Poly-*JOHN* consists of the points

$$J\begin{bmatrix}0\\3\end{bmatrix},\quad O\begin{bmatrix}0\\0\end{bmatrix},\quad H\begin{bmatrix}3\\0\end{bmatrix},\text{ and }\quad N\begin{bmatrix}3\\3\end{bmatrix}.$$

Plot these points in the plane and connect them to make *JOHN*.

(x) Multiply

$$\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$$

with each point in *JOHN* and draw a new polygon, called long-*JOHN*, on the same axes.

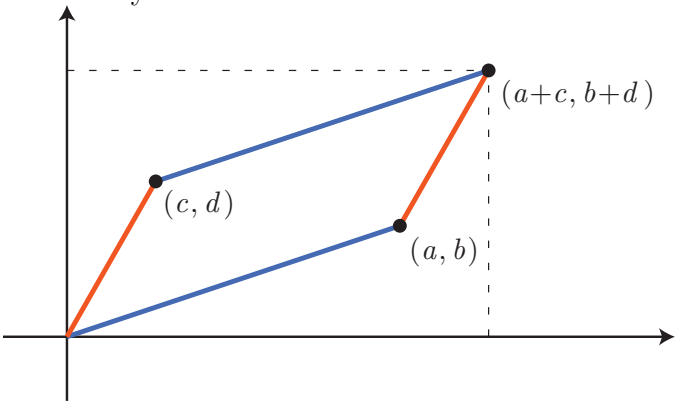
(y) What is the area of the original poly-*JOHN*?

Papa, I know you're upset because I was always your little square. But I've gotta transform!

Make *JOHN* what? Make *JOHN* drink a TaB?

And I've made up my mind, I'm keepin' my origin. . . ooh, gonna keep my origin. . .

1. Find the area of this parallelogram. Do not move along until you talk to someone else at your table who did this a different way.



2. What is the area of long-*JOHN*?

Long-*JOHN* is not silver and does not sell fish. Nor does he live in a pineapple under the sea.

3. Poly-*TOPE* is the parallelogram consisting of the points

$$T\begin{bmatrix} 1 \\ 5 \end{bmatrix}, \quad O\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \text{ and } \quad E\begin{bmatrix} 2 \\ 7 \end{bmatrix}.$$

(a) Plot these points in the plane and connect them to make *TOPE*.

(b) Multiply

$$\begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix}$$

with each point in *TOPE* and draw a new polygon, called iso-*TOPE*, on the same axes.

(c) Perform the transformation again to get taupe-*TOPE*. No need to plot these. What do you notice about the coordinates?

The one you wa-arned me about... he is a parallelogram... we're in an awful mess...

Taupe-*TOPE* is dope? Nope.

4. Find all scaled vectors for the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and determine their scaling factors. In other words, solve this system of equations:

$$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix}$$

5. Poly-*GLOT* is the parallelogram consisting of the points

$$G\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad L\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad O\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \quad T\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Plot these points in the plane and connect them to make *GLOT*. Now multiply

$$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

with each point in *GLOT* and draw a new polygon, called mono-*GLOT*, on the same axes. WOW what happened!? How does this relate to problem 4?

We're all glottons for punishment today.

Despite use of the capital letters, this problem is not affiliated with Blizzard Entertainment or Leeroy Jenkins.

6. Consider the recurrence $f(n) = 7f(n-1) - 10f(n-2)$.

(a) Let $g(n) = f(n) - 5f(n-1)$. Show that $g(n) = 2g(n-1)$ and therefore $g(n) = A \cdot 2^n$.

(b) Let $h(n) = f(n) - 2f(n-1)$. Show that $h(n)$ equals something important too.

(c) Solve this system for $f(n)$:

$$\begin{aligned} f(n) - 5f(n-1) &= A \cdot 2^n \\ f(n) - 2f(n-1) &= B \cdot 5^n \end{aligned}$$

Huzzah?

Neat Stuff

7. Remember the problem in the box?

(a) Keep multiplying poly-*JOHN* by the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ to get long-*JOHN*, big-*JOHN*, and big-bad-*JOHN*.

(b) What is the area of poly-*JOHN*? long-*JOHN*? big-*JOHN*? big-bad-*JOHN*?

How much does a large order of Fibonachos cost? It's the price of a medium order plus the price of a small order.

8. (a) What is the area of poly-*GLOT*? mono-*GLOT*?

(b) Describe how to tell that the area of mono-*GLOT* must be a specific multiple of the area of poly-*GLOT*. The graph from problem 5 should help.

9. Poly-*GAMY* is the parallelogram consisting of the points... aw, you just pick some points this time, okay? Plot these points in the plane and connect them to make poly-*GAMY*. Multiply

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

with each point in *GAMY* and draw a new polygon, called mono-*GAMY*, on the same axes. What do you notice?

All those points "got together", I guess. Manah-gamy... do doo do doo do.

10. Let $M = \begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix}$ as before. For each vector X, determine MX, M^2X, M^3X, and $M^{10}X$ without a calculator.

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$

(e) $4\begin{bmatrix} 1 \\ 5 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(f) $\begin{bmatrix} a \\ b \end{bmatrix}$

11. (a) Find the polygon "papa-*JOHN*", which is the predecessor of poly-*JOHN*.
(b) Find a matrix B that you can multiply poly-*JOHN* to get papa-*JOHN*.
(c) Find a matrix B that you can multiply long-*JOHN* to get poly-*JOHN*.

12. Here's an interesting transformation matrix:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

(a) Start with the point $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$ and keep multiplying it with A. What happens?
(b) Start with the point $\begin{bmatrix} -3 \\ 7 \end{bmatrix}$ instead. What happens?
(c) What's going on here? What happens to the corresponding recurrence?

Really though, aren't they *all* interesting? No. They are not.

13. Determine a rule for the area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$. Bonus points for using mattresses.

Please do not sing "Jump" while bouncing on matrices.

Tough Stuff

14. The *Tribbiani sequence* is defined by the recurrence

$$f(n) = f(n-1) + f(n-2) + f(n-3)$$

with three starting values. If you start with any set of integers, the ratio of consecutive elements does... what?

How *you* doin'? Yeah, don't start with all zeros, know what I'm sayin'? Sadly the inventor of this sequence is unable to count high enough to use it, but he is a mento for kids.

15. Mess around with the powers of the matrix

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Whatchu talkin' bout?

16. What's up with the recurrence $f(n) = 3f(n-1) - 3f(n-2) + f(n-3)$? Quadrophenia!

17. How long is the Fibonacci sequence in mod 19? What about the Tribbiani sequence?

What's yellow and equivalent to the Axiom of Choice? Zorn's Lemon! Get it? Neither did we. Bowen had a math book with a typo and for years thought it was called the Axiom of Choie.