13 *Vector of Light*

Important Stuff

PROBLEM

Poly-$WONA$ consists of the points

$$W\begin{bmatrix}0\\1\end{bmatrix},\quad O\begin{bmatrix}0\\0\end{bmatrix},\quad N\begin{bmatrix}1\\0\end{bmatrix},\text{ and}\quad A\begin{bmatrix}1\\1\end{bmatrix}.$$

Plot these points in the plane and connect them to make $WONA$.

(j) All Amanda wants is to multiply

$$\begin{bmatrix}5 & 1\\3 & 3\end{bmatrix}$$

with each point in $WONA$ and draw a new polygon, called alli-$WONA$, on the same axes.

(i) What is the area of the original poly-$WONA$ and the new alli-$WONA$?

(m) Find all the scaled vectors for the matrix

$$\begin{bmatrix}5 & 1\\3 & 3\end{bmatrix}$$

and the scaling factors for each. Do this by solving

$$\begin{bmatrix}5 & 1\\3 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}kx\\ky\end{bmatrix}$$

Poly-*WONA* cracker? Hey Mr. DJ, put a record on, I *WONA* dance with my baby. . .

Wona Fanta, don't you wanna. . .

All I *WONA* do is have some fun, until the sun comes up over Kearns Boulevard. . .

Well do ya, do ya do ya *WONA*. . .

1. Repeat the problem in the box using roly-$POLY$ and super-$POLY$ with the same matrix and the points

$$P\begin{bmatrix}-1\\3\end{bmatrix},\quad O\begin{bmatrix}0\\0\end{bmatrix},\quad L\begin{bmatrix}2\\2\end{bmatrix},\text{ and}\quad Y\begin{bmatrix}1\\5\end{bmatrix}.$$

Ooh, pretty. Jerry wants to know: how many times larger is super-$POLY$ than roly-$POLY$? Show it!!

Super Polident gets tough stains super clean. Ask Martha Raye, denture wearer.

Show me the scaling!!!!

2. Let $T(n)$ be defined by this tough-looking cookie:

Try this on the Nspire! It actually *works*! Pretty amazing.

$$T(n) = \begin{cases} \begin{bmatrix} 2 \\ 13 \end{bmatrix}, & n = 0 \\ \begin{bmatrix} 0 & 1 \\ -30 & 13 \end{bmatrix} T(n-1), & n > 0 \end{cases}$$

(b) Find $T(1)$, $T(2)$, $T(3)$ and $T(4)$.

(r) 13 and -30, eh? We saw these numbers in a box someday. For whence cameth this?

Faster than the speeding light she's BRAAAAA-WWWWWWWWWWWWWW

(i) What do the 0 and 1 in this matrix do, if the 13 and -30 are driving everything?

(a) Find a closed rule for $T(n)$. Your answer can involve some sort of matrix raised to some power, like n.

(n) Find all scaled vectors for the matrix $\begin{bmatrix} 0 & 1 \\ -30 & 13 \end{bmatrix}$.

3. Find A and B so that each of these is true.

This looks familiar, vaguely familiar. Almost unreal yet, it's like from a problem set...

(c) $A \begin{bmatrix} 1 \\ 3 \end{bmatrix} + B \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 11 \\ 82 \end{bmatrix}$

(h) $A \begin{bmatrix} 1 \\ 3 \end{bmatrix} + B \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$

(r) $A \begin{bmatrix} 1 \\ 3 \end{bmatrix} + B \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(i) $A \begin{bmatrix} 1 \\ 3 \end{bmatrix} + B \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(s) Use the last two results to help with this one:

$$A \begin{bmatrix} 1 \\ 3 \end{bmatrix} + B \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Neat Stuff

5. Let $M = \begin{bmatrix} 0 & 1 \\ -30 & 13 \end{bmatrix}$. For each vector X, determine MX, M^2X, M^3X, and $M^{10}X$ without a calculator.

What's our reason for writing part (r) this way?

(s) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 13 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(r) $4\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 7\begin{bmatrix} 1 \\ 10 \end{bmatrix}$

(n) $\begin{bmatrix} 1 \\ 10 \end{bmatrix}$

(a) $\begin{bmatrix} x \\ y \end{bmatrix}$

8. Describe the effect each matrix below has when we multiply them with things in the plane. Oh, and there's combos!

Sadly not the pretzel-flavored kind. For (p), multiply stuff with F first to get Fx, then multiply that with E to get EFx. Does order of operations matter??

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $D = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$

(e) $E = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(f) $F = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(p) F first, then E
(h) E first, then F
(u) C first, then D
(o) C first, then E
(n) E first, then C
(g) C^3

She's got herself a little inverse matrix... multiplies, and all she gets is one... ohh... ohh... one.

Shear-ish the love... every day, every joy...

13. You can think of a parallelogram as the shape spanned by two vectors from the origin: (a, c) and (b, d). In three dimensions, a *parallelepiped* is the shape spanned by three vectors from the origin: (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3). Find its volume in terms of the nine variables, and explain your work. Please try to do this without relying on a formula. (See picture on the next page.)

This could be the best math term ever. Parallelepiped! Say it three times and maybe Michael Keaton will come out of the box.

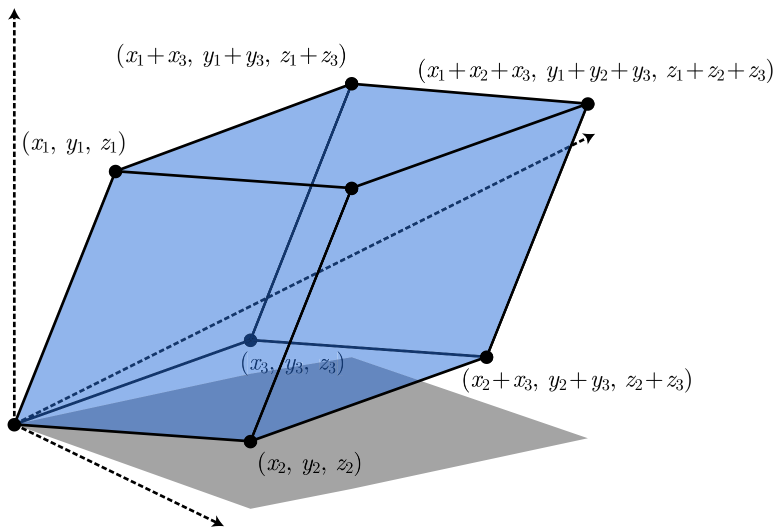

Figure adapted from `http://en.wikipedia.org/wiki/File:Determinant_parallelepiped.svg`.

21. Find all the scaled vectors for each matrix.

(t) $T = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

(i) $I = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix}$

(m) $M = \begin{bmatrix} 0 & 1 \\ -25 & 10 \end{bmatrix}$

34. Jessica challenges you to build a matrix whose only scaled vectors are along the line $y = 10x$. Do it!

55. Start with the matrix $X = \begin{bmatrix} b \\ m+b \end{bmatrix}$ and calculate $T^n X$ (where T is the matrix from problem 21) until you see what is happening. Wack.

Wiggida wack. Some of them try to rhyme, but they BRAAAAA-WWWWWWWWWWWWWW

89. Consider the recurrence $f(n) = 10f(n-1) - 25f(n-2)$. Try substituting $g(n) = f(n) - 5f(n-1)$ and see what develops.

144. Find all scaled vectors for this 3-by-3 matrix.

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 30 & -31 & 10 \end{bmatrix}$$

These Fibonachos are really tasty, and if you buy the right number of them, they come in a perfect square box.

233. Let

$$f(n) = \begin{cases} 0, & n = 0 \\ 5, & n = 1 \\ 37, & n = 2 \\ 10f(n-1) - 31f(n-2) + 30f(n-3), & n > 2 \end{cases}$$

Find a closed rule for $f(n)$.

Tough Stuff

377. For $n > 1$, can the sum of the first n squares ever be a perfect square? What about the sum of the first n perfect cubes?

Quicker than a vector of light and *gone!* Oh, and for the record, Ace of Base sucks.