14 *Crazy for $U(n)$*

Important Stuff

PROBLEM

(k) Find a closed rule for this recurrence:

$$U(n) = \begin{cases} 2, & n = 0 \\ 1, & n = 1 \\ U(n-1) + 6U(n-2), & n > 1 \end{cases}$$

(r) Find all the scaled vectors for the matrix

$$\begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}$$

and the scaling factor for each.

(i) Parallelogram *NORM* has the following vertices:

$$N\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad O\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad R\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \text{ and } \quad M\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Plot these points in the plane and connect them to form *NORM*.

(s) Multiply the matrix

$$\begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}$$

with each point in *NORM* to get nom-nom-*NORM*, and draw it and the original *NORM* on the same axes.

(t) What are the areas of the original *NORM* and nom-nom-*NORM*? What does our magic formula say about the area of nom-nom-*NORM* compared to *NORM*? Interesting.

(*i*) Multiply the same matrix with each point in nom-nom-*NORM* to make infinity-*NORM*. Plot infinity-*NORM* on the same axes as the others. *COOL!*

This problem is so much $f(U(n))$!! We're trying hard to control our hearts.

See yesterday's box if you are unsure what to do. If you're having trouble, we'll walk over to where you are.

Rumor suggests there may be a power outage at 1 pm today. Expect strangers making the most of the dark.

What kind of animal is Cookie Monster? An om-nom-vore.

The infinity norm is also known as the Chebyshev norm, the supremum norm, or sometimes the word "bigger". The infinity norm is often used to indicate a marathon of "Cheers".

1. Solve our CAPTCHA! (Wave for us to bring it to you.)

We'll see you through the smoky air.

2. Find A and B so that each of these is true.

(k) $A\begin{bmatrix}1\\3\end{bmatrix}+B\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}1\\13\end{bmatrix}$

(r) $A\begin{bmatrix}1\\3\end{bmatrix}+B\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

(i) $A\begin{bmatrix}1\\3\end{bmatrix}+B\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$

(s) $A\begin{bmatrix}1\\3\end{bmatrix}+B\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

This all breaks down to simple mathematics. 83% of these problems are extreme madness, add 2% gourmet falling cakes, divide by the month of August. Times f, multiplied by this drawing of a tiger, T minus that one thing, and the waterfall times muscle equals 4. (What?! Taken from `http://www.youtube.com/watch?v=pKI-hD49FnQ`)

3. Let $M=\begin{bmatrix}0&1\\6&1\end{bmatrix}$. For each vector X, determine MX, M^2X, M^3X, and $M^{10}X$ without a calculator.

(m) $\begin{bmatrix}1\\3\end{bmatrix}$

(a) $\begin{bmatrix}1\\-2\end{bmatrix}$

(r) $\begin{bmatrix}1\\3\end{bmatrix}+\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$

(y) $\frac{2}{5}\begin{bmatrix}1\\3\end{bmatrix}+\frac{3}{5}\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

(A) $\frac{1}{5}\begin{bmatrix}1\\3\end{bmatrix}-\frac{1}{5}\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$

(ndrews) $\begin{bmatrix}x\\y\end{bmatrix}$

Hey! Put that calculator away. Earlier in this course we saw someone using a calculator to multiply by 5, and it was the most offensive thing we've seen in our years of teaching. And that includes an elementary school production of Hair.

4. Find a closed rule for this recurrence:

$$T(n)=\begin{cases}\begin{bmatrix}x\\y\end{bmatrix}, & n=0\\ \begin{bmatrix}0&1\\6&1\end{bmatrix}T(n-1), & n>0\end{cases}$$

Get back from break on time! If you are one minute late, we will go to the animal shelter and get you a kitty cat. We will let you fall in love with that kitty cat. And then on some dark cold night we will steal away into your home and punch you in the face.

5. Find a closed rule for this recurrence in terms of x and y.

$$U(n) = \begin{cases} x, & n = 0 \\ y, & n = 1 \\ U(n-1) + 6U(n-2), & n > 1 \end{cases}$$

Didn't we do this already? Sure, but there's a fresh perspective now that might be helpful. Besides, we're out of original ideas, and we're gonna leave constant reinvention to Madonna.

Review Your Stuff

6. Your table must write two math problems that we will use on tomorrow's problem set. Consider the topics of the course, and choose appropriate numbers, contexts, and difficulties so that the problems can be solved by people in this room. Please take no less than 15 minutes and no more than 20 minutes for this task, and work together as a table. Sidenote jokes or references are welcome within reason and will be used if we decide they are actually funny.

We reserve the right to reject or edit any problem, primarily based on duplication between tables, or just because we feel like it. Wah wahhhhh.

Also we apologize about the sidenotes; normally "funny" isn't our criterion for selecting jokes.

Neat Stuff

7. Repeat the process of the Important Stuff questions with the matrix

$$M = \begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix}$$

Eventually, determine a closed rule for $M^n \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ for any vector, but use the road markers built up instead of trying to find a closed rule immediately.

One difference this time is that all the scaled factors have positive k. What's your $M(A(n))$ got to do with me? I got a $M(A(n))$.

8. An important matrix is

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

(a) Graph $NORM$, $M \cdot NORM$, and $M^2 \cdot NORM$. Which of these three parallelograms has the largest area?

(b) What are the scaled vectors and scaling factors of M?

(c) Describe what the graph of $M^n \cdot NORM$ will look like for a large value of n.

Ba ba ba ba ba, M *NORM NORM*, M *NORM*-a-*NORM*...

9. Find all the values of k so that the matrix

$$M = \begin{bmatrix} 0 & 1 \\ -30 & 13 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

produces shapes with no area when using M for transformations.

This problem is a little k00ky.

10. Rebecca challenges you to find all the scaled vectors and scaling factors for

$$M = \begin{bmatrix} 0 & 1 \\ -100 & 20 \end{bmatrix}$$

Oh noes! Stop asking tricky questions, Rebecca.

11. Consider the recurrence

$$f(n) = 20f(n-1) - 100f(n-2)$$

(v) Let $g(n) = f(n) - 10f(n-1)$. Show that $g(n)$ is exponential with the rule $g(n) = A \cdot 10^n$.
(i) Show that $f(n) = 10f(n-1) + A \cdot 10^n$.
(c) Write a rule for $f(n)$ in terms of $f(n-2)$... in terms of $f(n-3)$... in terms of $f(n-4)$.
(k) Normally a geometric series would be showing up right about now, but what is happening instead? Use this to find a closed rule for $f(n)$.
(i) Determine the particular solution for $f(0) = 0$ and $f(1) = 20$.

Today's teaching tip: empower your students to live in fear by creating an environment of irrational, random terror. Children need to be terrified. Stomp that yard! This tip brought to you by Sue Sylvester.

12. (a) Factor $A^3 - 15A^2 + 75A - 125$.
(b) Use the style of problem 11 to find a closed rule for the recurrence

$$f(n) = 15f(n-1) - 75f(n-2) + 125f(n-3)$$

$$f(0) = 1, f(1) = 15, f(2) = 175$$

Why was $f(n-3)$ afraid of $f(n-2)$? Because $f(n-2)$ $f(n-1)$ $f(n)$! Ha ha ha!!!

13. What happens if you work your matrix-multiplying magic on $x^2 + y^2 = 1$ (the equation for the unit circle) instead of a square or parallelogram? Try a few different matrices and see what happens.

Congratulations to Darryl on being voted the Most Chill Teacher of 2010! Woo!

Tough Stuff

14. Find all possible values of b and c such that $x^2 + bx + c$ and $x^2 + bx - c$ are both factorable over the integers.

We think Lance Armstrong could really benefit from the results of the Banach-Tarski paradox.