Problem Set 2: Slhi'unfg

Today's title comes from the German word for "shuffling".

Opener

Can perfect shuffles restore a deck with 9 cards to its original state? If so, how many perfect shuffles does it take? If not, why not?

Split the cards 5-and-4, and keep the top card on top.

Important Stuff

1. Working with your table, fill in a whole lot of *this* table:
`http://www.tinyurl.com/perfectshuffle`

The file is *in* the computer! Oh, only one computer per table, please.

2. Find the units digit of each annoying calculation. Put those calculators away!

a. $2314 \cdot 426 + 573 \cdot 234$
b. $(46+1)(46+2)(46+3)(46+4)(46+5)$
c. $71^4 \cdot 73^4 \cdot 77^4 \cdot 79^4$

The *units digit* of 90210 is 0, matching Brenda's IQ.

You down with ZPP? (Yeah, you know me!)

3. Find all possible values for the units digit of each person's positive integer.

a. Amy: "When you add 5 to my number, it ends in a 2."
b. Brandon: "When you multiply my number by 3, it ends in a 7."
c. Carmen: "When you multiply my number by 6, it ends in a 4."
d. David: "When you multiply my number by 5, it ends in a 3. Yup."

4. Unlike "base 10", in *mod 10* the only numbers are the remainders when you divide by 10. In mod 10, 6 + 5 = 1 because 1 is the remainder when 6 + 5 is divided by 10.

Answer all these questions in mod 10.

a. $2+2=\square$
b. $3 \cdot 4 = \square$
c. $\square + 5 = 2$
d. $4 \cdot \square = 2$
e. $5 \cdot \square = 3$
f. $\square^4 = 1$

This is sometimes called *modular arithmetic*. Clock arithmetic is mod 12. Four hours from now, it will be four hours later than it is right now.

That last one says box to the fourth power, by the way.

5. Repeat the previous problem, except this time do the arithmetic in *mod 7* instead of mod 10.

Good news: there are only 7 numbers in mod 7. Bad news: in mod 7, every Monday is the same.

6. Go back to the big table that we all filled in together. What patterns do you notice?

Some examples of bad patterns:

"I noticed that most of the values in the table were numbers."

"All the digits in the table could also be found on a computer keyboard."

"Some of the numbers in the table were bigger than others, while others were smaller."

"The leftmost column increased by 1 each time."

"Purple Gingham."

"New Mexico got to have the first two columns because they're in a different time zone."

We've provided some blank space below for you to write your own bad patterns . . .

Neat Stuff

7. Write each fraction as a base-10 decimal.

a. $\frac{1}{5}$ **e.** $\frac{2}{7}$

b. $\frac{1}{25}$ **f.** $\frac{6}{7}$

c. $\frac{1}{7}$ **g.** $\frac{1}{13}$

d. $\frac{3}{7}$ **h.** $\frac{2}{13}$

8. Write each base-10 fraction as a base-3 decimal. Some of the answers are already given, in which case—awesome!

a. $\frac{1}{13} = 0.\overline{002}_3$ **f.** $\frac{1}{7} = 0.\overline{010212}_3$

b. $\frac{2}{13}$ **g.** $\frac{3}{7}$

c. $\frac{3}{13}$ **h.** $\frac{9}{7}$

d. $\frac{9}{13}$ **i.** $\frac{6}{7}$

e. $\frac{10}{13}$

9. Write the base-10 decimal expansion of

$$\frac{1}{142857}$$

10. Marvin wonders what kinds of behavior can happen with the base-10 decimal expansion of $\frac{1}{n}$. Be as specific as possible!

11. If $\frac{1}{n}$ terminates in base 10, explain how you could determine the length of the decimal based on n, without doing any long division.

You now know the entire plot of the horrible movie *Terminator 1/4: 0.25 Day*.

12. Robyn wonders what kinds of behavior can happen with the base-3 decimal expansion of $\frac{1}{n}$.

13. We overheard Sara and Joe still yelling about whether or not the number .99999... was equal to 1. Is it? Be convincing.

A little ditty, bout Sara and Joe. Two mathematical kids doin' the best that they know.

14. **a.** Suppose $ab = 0$ in mod 10. What does this tell you about a and b?

b. Suppose $cd = 0$ in mod 7. What does this tell you about c and d?

It tells you that a through d hog the spotlight too much. No love for the middle of the alphabet in algebra.

15. Investigate shuffling decks of cards into three piles instead of two. What are the options? Does it "work" like it does with two piles?

16. **a.** Investigate the base-10 decimal expansions of $\frac{n}{41}$ for different choices of n. What happens?

b. Investigate the *base-3* expansions of $\frac{n}{41}$ for different choices of n. What happens?

The fraction $\frac{n}{41}$ is still in base 10 here, so don't convert 41 to some other number.

17. **a.** Find all positive integers n so that the base-10 decimal expansion of $\frac{1}{n}$ repeats in exactly 4 digits.

b. Find all positive integers n so that the base-3 "decimal" expansion of $\frac{1}{n}$ repeats in exactly 5 digits.

18. Write 223 and 15.125 in base 2. Then write them in base $\sqrt{2}$. How cool is that?!

While this problem is cooler than most math, the Supreme Court recently ruled that math cannot actually be cool.

Tough Stuff

19. Aziz has a cube, and he wants to color its faces with two different colors. How many different colorings are possible? By "different" we mean that you can't make one look like the other through a re-orientation.

20. Barbara has an octahedron, and she wants to color its vertices with two different colors. How many different colorings are possible? By "different" we mean that you can't make one look like the other through a re-orientation.

21. What about edges?

Edges? Edges? We don't need no stinkin' edges!

22. Find all solutions to $x^2 - 6x + 8 = 0$ in mod 105 without use of any technology. There's probably more.