Day 6 (July 10, 2012)

8. Donna asks what size decks will get restored to their original order after exactly 10 Thursday-style shuffles (and not in any fewer number of shuffles).

We know that a 50-card deck is restored after 8 Thursday-style shuffles because

$$2^8 = 1 \mod 51.$$

Another way to say this is

$$2^8 - 1 = 255 = 0 \mod 51$$

$$255 = 5 \times 51$$

What size decks are restored in 3 Thursday-style shuffles?

$2^3 = 1 \mod n$ for what n?

$2^3 - 1 = 0 \mod n$ for what n?

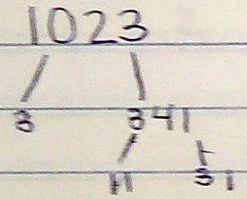
$2^3 - 1 = 7 \rightarrow$ only factors are 1 & 7

So a 6-card deck is restored after 3 Thursday-style shuffles, and that's the only size deck that will do so!

Donna's solution for which decks restore after 10 Thursday-style shuffles:

10 shuffles

$2^{10} - 1 = 1023$



# of cards

3 but not because $3 < 10$

$3 \cdot 11 = 33$ and 34

31 and 32

but not because $32 \equiv 0 \bmod 2$

11 and 12

$11 \cdot 31 = 341$ and 342

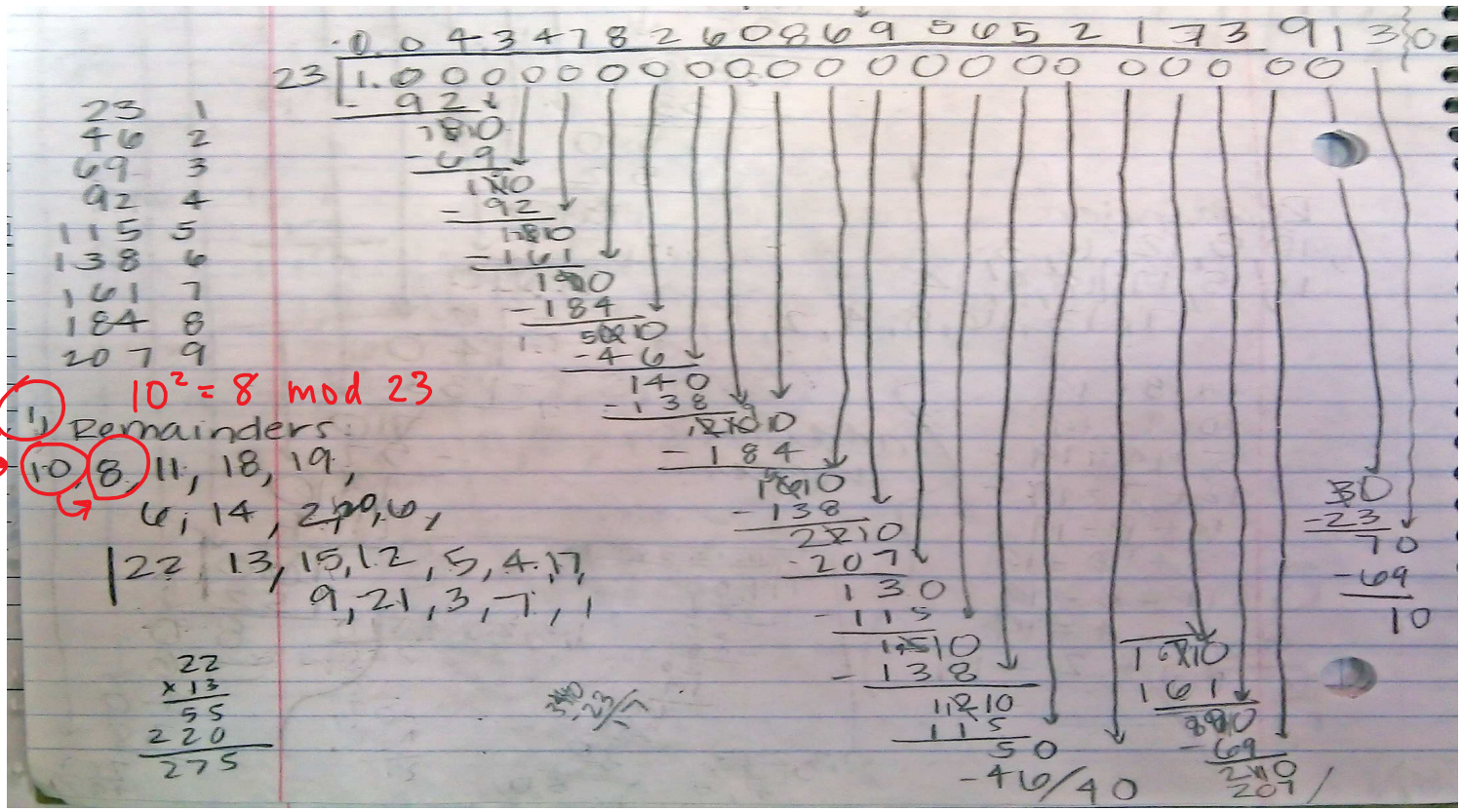
1023 + 1024

$1023 = 1024 - 1 = (2^5)^2 - 1$

$= (2^5 - 1)(2^5 + 1)$

$= 31 \cdot 33 = 31 \cdot 3 \cdot 11$

Jen's observation about remainders when calculating 1/23



The remainders are the powers of 10 in mod 23!