

Problem Set 8: Miscellaneous

Opener

What do these equations have to do with the base 2 “decimal” for $\frac{1}{21}$?

$$1 = 0 \cdot 21 + 1$$

$$2 = 0 \cdot 21 + 2$$

$$4 = 0 \cdot 21 +$$

$$8 = \quad \cdot 21 + 8$$

$$16 = 0 \cdot 21 +$$

$$32 = 1 \cdot 21 +$$

$$22 = 1 \cdot 21 +$$

from *Hollywood Shuffle*:
Ain't nothin' to it, but to do it!
This movie featured Keenen
Ivory Wayans as “Jheri Curl”
(he also wrote the movie).

Finish the equations above.

Important Stuff

1. Complete this table.

n	“Decimal” for $1/n$ in base 2	# of repeating digits	# of units in mod n
3	$0.\overline{01}$	2	2
5			
7			
9			
11			
13			
15			
17			
19			
21	$0.\overline{000011}$	6	

Using the method from the opener may be helpful to your sanity, but you can also use long division. Split the work, but keep away from technology or you may miss some big ideas.

The *Ickey Shuffle* is the most famous touchdown dance of all time. One of the two TV announcers from the movie *Cars* did the Ickey Shuffle after winning a NASCAR race. For more information, watch the “Return of the Shirt” episode from *How I Met Your Mother*. It's . . . wait for it . . .

2.
 - a. Write out all the units in mod 21.
 - b. Write out all the powers of 10 in mod 21, starting with 1 and 10.
 - c. Take all the powers of 10 in mod 21 and multiply them by 2. What happens?
 - d. Take all the powers of 10 in mod 21 and multiply them by 3. What happens?
 - e. Take all the powers of 10 in mod 21 and multiply them by 7. What happens?

... legendary!!

Hey, you're still in mod 21!
There's no such number as 32.

3. Find each decimal expansion in *base 10*. Seek shortcuts to simplify your work!

Fraction	Decimal	Fraction	Decimal
1/21	0.047619	11/21	
2/21		12/21	
3/21		13/21	
4/21		14/21	
5/21		15/21	
6/21		16/21	
7/21		17/21	
8/21		18/21	
9/21		19/21	
10/21		20/21	

If you use a calculator, report your answers as repeating decimals instead of rounding them off.

Ali: "Now hold it. The *Ali Shuffle* is a dance that will make you scuffle. During the time that I'm doing this shuffle, for a minute, you're going to be confused. You must get in a boxing position, and have a little dance." Howard Cosell – do the voice: "What we've just seen perhaps is the heavyweight champion of the world in what should be his true profession, that of a professional dancer."

4.
 - a. How many fractions in the table above are in lowest terms?
 - b. How many units are there in mod 21?
 - c. Pick three different units in mod 21. For each, calculate u^{12} in mod 21.
5. Describe any patterns you notice in the table. What fractions form "cycles" that use the same numbers in the same order? Write out each cycle in order. How long are the cycles?

A fraction is in *lowest terms* if it cannot be reduced. 7/11 is in lowest terms, but 6/10 is not.

Coicles? I don't see any coicles here, nyuk nyuk nyuk.

6. Let's try it again, but this time we'll use *base 2*. Find each "decimal" expansion. Seek shortcuts to simplify your work.

Fraction	Base 2 "Decimal"
1/21	0.000011
2/21	
3/21	
4/21	
5/21	
6/21	
7/21	
8/21	
9/21	
10/21	

Fraction	Base 2 "Decimal"
11/21	
12/21	
13/21	
14/21	
15/21	
16/21	
17/21	
18/21	
19/21	
20/21	

The *Curly Shuffle* is a 7-cycle on the floor: "Wooooooooo woo woo woo, woo woo woo. Wooooooooo woo woo woo, woo woo woo." See also: Angus Young in Let There Be Rock; Homer Simpson in the "Lisa needs braces" episode.

7. Here is a 22-card deck under Monday-style shuffling.

<http://www.tinyurl.com/22cards>

Follow some cards and follow some remainders. How can you use the cards to find the *entire base-2 expansion* for each fraction?

Where? I don't see it. Oh, it's *in* the computer.

How can we be expected to teach children to learn how to read *if they can't even fit inside the building?*

Neat Stuff

8. So, shuffling. Thomas is so good at it that he suggests you get a deck whose size is a multiple of 3 and try triple-out-shuffling and triple-in-shuffling! Cut the cards into three piles then shuffle them together from either the left or the right.

See what you find. We recommend using the same card notation: with out-shuffles, count cards as 0, 1, 2, With in-shuffles, count cards as 1, 2, 3, There's a lot to find!

Yeah yeah yeah, shake a tail feather baby . . . Yeah yeah yeah, do the *Harlem Shuffle*. Thursday night is karaoke night . . . maybe this time. The director of Harlem Shuffle's music video created *Ren & Stimpy!*

Curly: I'm tryin' to think, but nothin' happens!

9. a. If you haven't yet, go back and do Problem 10 from Set 7.
 b. The *order* of a unit u in mod n is the smallest power $k > 0$ such that $u^k = 1$ in mod n . Prove that the order of any unit u in mod n must be a factor of the number of units in mod n .
10. a. OK, so about that "magic" thing we did back on Monday. How'd we do that?
 b. Here's a hint from Sousada: go back and do Problems 14 and 15 on Set 7 if you haven't already.
 c. Here's another hint from Soledad: look at the eight numbers in the table for the 6♠ row. Pretend those numbers are written after a "decimal point" and are repeating digits in a base-2 "decimal" expansion. What is the value of this repeating base-2 "decimal"?
11. The decimal expansion of $\frac{1}{7}$ is $0.\overline{142857}$. Now, split the repeating digits in thirds and add them together:

$$14 + 28 + 57 = 99.$$

Try this with other fractions $\frac{1}{n}$ whose repeating digit lengths are multiples of 3. What's up with that!

12. Suppose a and b are relatively prime. How does the length of the decimal expansion of $\frac{1}{ab}$ compare to the lengths of the decimal expansions of $\frac{1}{a}$ and $\frac{1}{b}$?
13. In a Reader Reflection in the *Mathematics Teacher* (March, 1997), Walt Levisee reports on a nine-year-old student David Cole who conjectured that if the period of the base-10 expansion of $\frac{1}{n}$ is $n - 1$, then n is prime. Prove David's conjecture, and discuss whether it might apply to other bases.

Tough Stuff

14. Prove that if p is an odd prime, there is at least one base $b < p$ in which the expansion of $\frac{1}{p}$ has period $p - 1$. Bonus: determine the number of such bases in terms of p .

You've got to go back Marty!
 Back to . . . oh, the past.
 Bah.

Anarchy in the u^k !

You've got to go back Marty!
Shut up. Oh, and if you see anyone claiming that today is the day from *Back to the Future II*, tell them to shut up, because they've fallen for a bad Photoshop for like the fifth time. The real future date is October 21, 2015, and it predicted all sorts of ridiculous crap like Miami being in the World Series, wall-mounted TVs that would show multiple channels at once, ridiculous numbers of 3D movie sequels, video games you could control with your hands, and video conferencing. Oh.