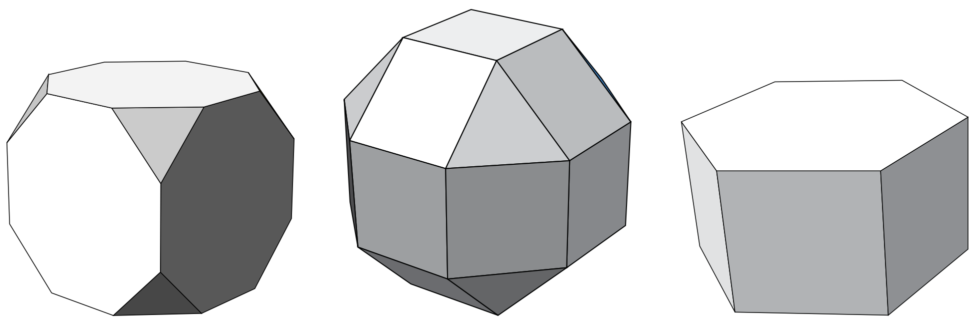
Problem Set 14: Some Math Camp That I Used To Know

Opener

Fill in this table.

Mwah ahaha! You thought you were done with tables?

Polyhedron	Faces (F)	Edges (E)	Vertices (V)	$F - E + V$
Cube	6	12		
Tetrahedron	4			
Truncated Cube				
Rhombicuboctahedron				
Hexagonal Prism				



Truncated Cube Rhombicuboctahedron Hexagonal Prism

Please be careful, and do not hit Ashli in the head with any more solids.

Important Stuff

1. For integers $a, b \geqslant 3$, find all possible solutions to the inequality

$$\frac{1}{a} + \frac{1}{b} > \frac{1}{2}$$

2. **a.** How many orientations does a cube have? Call this number n.

   b. Calculate each piece along with the total value of

$$\frac{n}{4} - \frac{n}{2} + \frac{n}{3}$$

   What do you notice?

But you didn't have to come to Utah
Meet some friends and shuffle cards and then mod some numbers
I guess you've got to leave us though
Now we're just some math camp that you used to know

3. A *Platonic solid* is made of regular polygons that meet the same way at all vertices. Suppose a Platonic solid has faces with a sides and vertices where b edges meet.

But you say it's just a friend.

- **a.** If n is the number of orientations of this mystery solid, find the number of faces, edges, and vertices in terms of n.
- **b.** Rewrite this equation so that one side says $\frac{1}{a} + \frac{1}{b}$.
- **c.** Find all possible solutions to the equation, along with the value of n for each. Oh snap.
- **d.** How many Platonic solids are there, and what are the options?

Ooh, it's a mystery solid! Intriguing. Do tell more.

Your Stuff

NM. **a.** Complete this equation in base 8.

$$0.\overline{7202013} + \boxed{} = 1$$

What will Darryl wear to Niagara Falls if he decides to become a stuntman? A *Darryl barrel*!

- **b.** List all the powers of 2 in mod 1025 using only a four function calculator.
- **c.** Find the base-5 decimal for 8/15.

2. Divide 20 by 7 and leave your answer in base 2.

Because it's an alien language, Jay knows!

10. Convert these base-10 numbers to base 5 now now now!

a. 73 **b.** $\frac{1}{25}$ **c.** $\frac{1}{2}$

NOW!

6.
- **a.** Prove $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ using decimal expansions in base 10.
- **b.** Prove $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ using decimal expansions in base 3.
- **c.** Prove $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ using decimal expansions in base 2.
- **d.** Compare the decimal expansion of $\frac{1}{3}$ in each of the three bases. What do you notice?

Only 3 of your base are belong to us! (Great joke, Kieran! You really had that one pegged.)

5. Andrea has an 8-card deck for some in-shuffling.

- **a.** Track the location of each card. What cycles do you notice?
- **b.** Armando has a 14-card deck. Repeat! What do you notice?

c. Go back to your 8-card deck and write out the order of all 8 cards after each in-shuffle. Explain the following observation.

$$1 \cdot 5 = 5 \bmod 9$$
$$5 \cdot 5 = 7 \bmod 9$$
$$7 \cdot 5 = 8 \bmod 9$$
$$\vdots$$

d. Does this pattern work for other deck sizes?

e. How can you use this pattern to determine the number of in-shuffles that it will take to restore an n-card deck?

Now and then I think of all the times you gave me Neat Stuff
But had me believing it was always something I could do
Yeah I wanna live that way
Reading the dumb jokes you'd play
But now you've got to let us go
And we're leaving from a math camp that you used to know

12. How many in-shuffles does it take to restore a 12-card deck? What if you cut the deck into 3 piles? 4 piles? 6 piles? 5 piles?!

1 & 9. Suppose that while performing out-shuffles on a regular 52-card deck, you track whether a particular card appears on the left (L) or right (R) of the cut and you record that information.

a. If you notice that the card goes LLLRRLLR, what card is it?

b. Repeat for LRLLRLLL.

c. The person writing the pattern for that last one might have messed up the last two letters. What should the last two letters be, and what card is it?

High five, you're a STaR!

d. Repeat for RLLRLR??.

e. Repeat for RRLLLR??.

3. Dowen and Barryl are avid Euchre players, so they want to try out their "card prediction" trick from last Monday using a 24-card Euchre deck (9 through Ace of each suit) instead of a 52-card deck.

MOAR BARREL

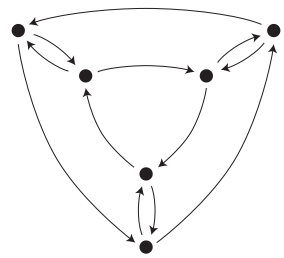
a. How many out-shuffles will it take to restore the deck to its original position?

b. What is 1/23 represented as a base-2 decimal?

c. What card positions does the 5th card take before it returns to its original position?

d. How many "Left-Rights" would you need to be told in order to predict the exact card that was chosen?

8. In what contexts has this graph come up during this class?



Rejected question: "Create an isomorphism between something we learned in class and any Georgia O'Keefe painting of your choice."

2. True or false: for all regular polygons, the number of symmetries/orientations is equal to twice the number of edges.

2. Make the most obnoxious table *evar*: an operation table for *all* the transformations of the cube, where the operation is "then."

Let's just be crystal clear we did not write this problem. Table 2 is asking you to do this. Not us.

11. **a.** Deborah loved Problem 3 from Set 12 so much that she redrew the graph using blue arrows for the in-shuffles and yellow arrows for the out-shuffles. Where have you seen a picture like hers before?

b. Gabe suggests making another picture. He says to draw the integers 0, 1, 2, 3, 4, an 5 and use yellow arrows to to connect two integers if you get from the first integer to the second by adding 3 (mod 6). Use blue arrows to connect two integers if you get from the first integer to the second by adding 2 (mod 6). Where have you seen a picture like his before?

Picture Gabe on stage singing some Beyonce. Better yet, don't.

c. Peter says something was missing from our parade construction. What was missing? Barb thinks this must be why we were getting confused looks from the crowd.

Put them together and what do you get? Bibbity boppity group. Symmetric across a Z La sub3 group la group. (Uh, *what*?)

12. Oh *noes*! A feared red math germ has begun infecting PCMIers! Kieran suggests that it might be possible to come up with a cure by shuffling the red math germ's genes to create an antidote. He lists the genes as "r e d m a t h g e r m" and then transforms its genes using (1 2)(3 11)(10 9 8 7 6). What do you think of Kieran's suggestion?

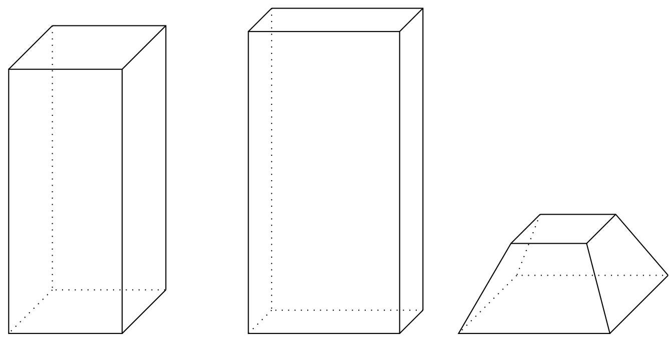
Really, Table 12, really? *Really?* I guess it's true, we can't stop you.

Hey! This is math camp, not virology camp!

10. Refer back to Problem 1 from Set 13. Find two counterexamples to the statement "You can choose any two transformations and always achieve all eight symmetries/orientations."

12. How many different transformations can you perform on these non-platonic solids, while still making sure that they occupy the same volume of space?

If they're non-platonic, does that mean these solids hooked up? Zig a zeg ahh.



rectangular prism (square base, not a cube) — monolith ($L \neq W \neq H$) — frustum (square base, see a \$1 bill)

Don't let these problems frustumrate you!

7. **a.** If you number a 52-card deck from 0 to 51 and track the position of card #1 during out-shuffles, you will get this cycle: $S = \{1, 2, 4, 8, 16, 32, 13, 26\}$. Make an operation table for these numbers, where the operation is multiplication mod 51.

b. Show that this group is isomorphic to $(\mathbb{Z}_8, +)$.

c. Choose a different cycle of cards by tracking a different card as the deck is out-shuffled. Does that set of numbers also form a group under multiplication mod 51?

This problem is as awesome as Fred's name tag. It's like the Bohemian Rhapsody of math. Everybody knows that song!

11. **a.** Put the numbers $1, \ldots, 12$ on the edges of a cube so that the four numbers around each face have the same sum.

b. Find 19 other really different ways to do this, where "really different" means not a cube symmetry of another solution. Hint: Everybody's shuffling.

Our Stuff

5. Show that S_4 is isomorphic to the cube by finding four interesting axes of symmetry.

4. Show that a transposition cannot be both even and odd at the same time.

3. **a.** How many *distinct* ways are there to number the four faces of a d4? By this we mean that you can't find a symmetry that brings one to another. Wow, that's not many!

b. How about for a d6?

c. How about for a d8? Wow, that's . . . not not many?

2. The number of orientations of a d12 and a d20 is the same. Are these isomorphic doyathink?

1. Figure out how to use shuffling to find the *base-10* decimal expansion of $\frac{1}{51}$ along with other fractions.

No More Stuff

Balderdash time! Six of the seven definitions below are false. Which is the right one?

Richard says that the S in S_4 stands for *Schrödinger*, a man who owned too many cats.

Tina says that the S in S_4 stands for *scaler*, someone who climbs a large mountain in Park City.

Mark says that the S in S_4 stands for *subgroup*, a set of nuclear wessels.

Rina says that the S in S_4 stands for *set*, something you win at tennis by finding three cards with different properties.

Rebecca says that the S in S_4 stands for *simplex*, the ability to make things easier and harder at the same time.

Robert says that the S in S_4 stands for *seecant*, someone who really needs glasses.

Jodie says that the D in S_4 stands for *symmetric*, meaning . . . you know, symmetric!

Don't you forget about us
We'll be alone, shufflin', you know it baby
These groups, we'll take them apart
Then put 'em back together in parts, baby

I say $(LA)^{55}$

When you walk on by

Will you call me maybe . . .

(See you again soon.)