Problem Set 2: Die Tossing, With A Vengeance

Opener

Here's a game with cash prizes! One lucky contestant has the chance to win an *unlimited* amount of money!

For $\mathbb{R}$!

In front of them is a very large number of standard dice. The contestant will pick a specific number of dice, then roll them all at once. The goal for the contestant is to avoid rolling a six.

Depending on context, "rolling a six" could be interpreted as "stopping at a Whammy" or "accidentally playing Nickelback".

If the contestant avoids a six, they earn **$1 for every die they rolled**. If the contestant rolls any sixes, they win nothing.

1. How many dice should the contestant pick?

Important Stuff

2. Describe, in complete detail, a test you could perform on a set of 120 coin flips that would help you decide whether it is real or fake. Try to revise or improve your test from Day 1.

3. *Use your test* on your choice of four of these data sets to decide whether they are real or fake.

In each of these, read from left to right across the rows. Set (a) begins with two heads, a tail, three heads, two tails, then eight heads in a row.

a.	1101110011	1111110000	1111111111
	1111111110	1110001111	1110000111
	0001111111	0001111100	1110010101
	1001001111	1100000111	0000011111
b.	1001001110	1011000011	0010011100
	1011100010	1011100111	1011100011
	1001101010	0010011011	0110101101
	0111001100	1100110110	0110010011
c.	0001011010	1001011111	0110110011
	0000000000	1000011110	0100100011
	1010100011	1111111010	0101100110
	1001011000	1111001110	1100101011

Why was 110 afraid of 111? Because 111 1000 1001.

d.

1101101000	1111110010	1010101100
1011001100	1001101011	1000110110
1011010010	0000100000	0111111000
0010001011	1011100000	0000000100

e.

0010010110	0111000110	1001101011
0100011010	1010001000	0100000101
0000111010	1111010001	0000011010
1011110111	0000010011	1111111111

f.

1100100011	1010111001	1111001010
1001110110	0011100000	1010101001
0011000101	1101011001	1001001001
1100001101	0011101101	1010011000

g.

0011010000	0101101101	0110010001
0110011100	1111110100	1100111101
0010011001	1000111010	1000111010
1011000111	0100000000	0111011001

There are 10 types of people in the world . . .

Bender: What a horrible dream. 1's and 0's everywhere, and I thought I saw a 2!

Fry: It's okay Bender, it was just a dream. There's no such thing as 2.

4. **a.** What is the probability of throwing four heads on four consecutive coin flips?

b. What is the probability of throwing four coins and having them all come up "the same"?

c. What is the probability of throwing 10 coins and having them all come up "the same"?

5. Yesterday we flipped a lot of coins. Estimate the largest consecutive run of heads or tails *anyone in the room* achieved during their coin-flipping, and explain why you came up with this estimate.

6. Find the probability that two positive integers do not share a common factor greater than 1, given that . . .

a. . . . one of the numbers is 1.

b. . . . one of the numbers is 3.

c. . . . one of the numbers is 5.

d. . . . one of the numbers is 9.

e. . . . one of the numbers is 6.

7. Find the probability that two integers between 1 and 10 (inclusive) have no common factor greater than 1. There is more than one way to do this: how do the possible answers compare?

Neat Stuff

8. **a.** Trang rolls four dice. What is the probability that she will *avoid* rolling a six on any of them?
b. Bryce also rolls four dice. What is the probability that he will *hit* at least one six?
c. Kitty rolls two dice. What is the probability that she will roll two sixes together?
d. Usha rolls two dice 24 times. What is the probability that she never rolls two sixes together?

9. Gail gives you a (potentially real or fake) list of 120 coin flips. Turns out, it has exactly 60 heads and 60 tails! Does this suggest the list is real or fake? How strong would you consider this evidence to be?

10. In 120 truly random coin flips, what should be the average number of "runs" of flips? For example, the following flipping sequence has 7 runs:

011100011010

You know what was truly random? That guy that was in the elevator in that *Gangnam Style* video!

11. When we last checked, there were 67 real and 65 fake data sets submitted via `http://bit.ly/fakeflips`. 15 of the 132 data sets have an exactly 60 heads and 60 tails, and 8 of these 15 are fake. Reconsider your answer to Problem 9 in light of this information.

12. 44 of the 132 data sets have no "runs" of 6 or longer, and 33 of the 44 are fake. If Jessica gives you a data set whose longest run is 5 or shorter, does this suggest her data set is real or fake? How strong would you consider this evidence to be?

Krylon Paint! No runs, no drips, no errors. Thanks, Johnny Bench!

13. On *Extreme Die Tossing* players are paid $1,000 multiplied by the total value of the dice they throw, as long as they avoid the pesky six. (In our version, the contestant could win $1 multiplied by the number of dice they throw.)

a. Why might the show use the six as the "bankrupting" number in the game?

b. Find the contestant's best strategy, and the amount of money the show should expect to give out per contestant.

This is an *extremely* boring show. It's almost as bad as a game show about bingo, but such a thing could never be on network television, right?

14. Brian, Brian, Marla, Jennifer, Moe, and Elmer go to dinner every night and play "credit card roulette": the waiter picks one of their six credit cards at random to pay for the meal. What is the *mean* number of meals it will take before each of them has paid at least once?

15. Jack is chosen to play a coin-flipping game, in which he gets $1 every time he flips heads. But, if he flips tails he is in "danger" and must flip heads next. If he flips tails twice in a row, he will "bust" and lose all his money (but continue playing). The game lasts 10 flips.

Perhaps Danger is Jack's middle name?

a. Find the probability that Jack survives all 10 flips without busting even once.

b. Determine the average amount of money Jack could expect after 10 flips.

c. What would happen in a longer game? Will the average payout increase or decrease?

Tough Stuff

16. On the game show "Tic Tac Dough", the bonus game consists of nine squares:

Two squares containing TIC and TAC
Six money squares worth 100, 150, 250, 300, 400, 500
The dreaded DRAGON, which ends the round

A little known fact: The dragon from "Tic Tac Dough" went on to star opposite Dennis Quaid in the movie *Dragonheart*.

The player wins if they can hit TIC and TAC, *or* if they can collect $1000 from the money squares, before hitting the dragon. Find the probability that the player wins the game.

17. In a set of 120 real coin flips, what is the probability of getting at least one "run" of 7 consecutive flips (either heads or tails)? What is the probability of a run of at least 8? 9?