Problem Set 3: Big Deal

Opener

On *Deal or No Deal*, contestants open suitcases with different dollar amounts in them. Every so often, the player is offered a deal to walk away instead of continuing the game. If the player refuses all deals, they get to keep whatever is in the final suitcase.

1. There are four suitcases left: \$5, \$25, \$10,000, and \$50,000. You're the contestant.

It's been a bad run, unfortunately.

a. The show offers you \$9,000 to walk away. Would you take the deal? Why or why not?

b. What if the show offered you \$12,000? Would you take the deal? Why or why not?

2. There are five suitcases left: \$1, \$5, \$5,000, \$30,000, and \$125,000. If you represented the show, what deal amount would you offer the contestant, and why?

3. The show begins with these 26 suitcase values:

\$.01	\$1	\$5	\$10	\$25	\$50
\$75	\$100	\$200	\$300	\$400	\$500
\$750	\$1,000	\$5,000	\$10,000	\$25,000	\$50,000
\$75,000	\$100,000	\$200,000	\$300,000	\$400,000	\$500,000
\$750,000	\$1,000,000				

Daytime versions and special episodes have different values, but these are the 26 suitcase amounts from the original U.S. version of *Deal or No Deal*.

What would a *fair deal* be worth at the start of the game? Describe how the calculation was made.

4. Carl has three suitcases left: \$1, \$5, and \$1,000,000. He's resolved already that he's going all the way and will refuse any deal. He's about to call off the last two other suitcases.

a. What is the probability that Carl wins the \$1,000,000?

b. What is the probability that Carl survives the first suitcase opening? By "survives" we mean he doesn't open the \$1,000,000 case.

c. Given that Carl survived the first suitcase opening, what is the probability that he also survives the second opening?

Important Stuff

Here, have some two-way tables!

	NY	The Burbs
went to pub trivia	10	8
did something more meaningful	15	45

	NY	Rest of Us
female	7	39
not female	18	14

5. One teacher is chosen at random from the roster here at PCMI. What is the probability that this person currently teaches in New York, given . . .

New York City?? Get a rope.

a. . . . no additional information?

b. . . . that the person went to pub trivia last night?

c. . . . that the person is female?

d. . . . that the person is named Kieran?

Note, we're saying the person is female, but didn't necessarily go to pub trivia last night.

Unhelpful hint: the probability that Kieran won last night's pub trivia is 0.

6. Aziz and Ben each choose one positive integers at random. What is the probability that their numbers share no common factor larger than 1, given that . . .

a. . . . Aziz's number is 2?

b. . . . Aziz's number is 2 *and* Ben's number ends in a 7?

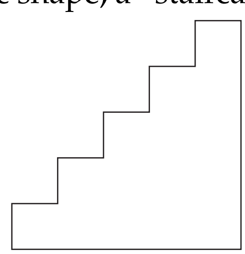
c. . . . Aziz's number is 2 *and* Ben's number ends in an 8?

d. . . . Aziz's number is 5?

e. . . . Aziz's number is 5 *and* Ben's number is a multiple of 3?

7. Here is a geometric shape, a "staircase" with five stairs:

And she's buying a staircase . . . to Problem 7 . . .



a. If the length and height of each stair is 1 foot, find the area of the shape.

b. Show how two staircases of this shape could be combined into a rectangle, then find the area of the rectangle.

c. Suppose the staircase had 9 steps instead of 5. Could two such staircases still form a rectangle? Use this to find the area of *one* staircase with 9 steps.

d. Suppose the staircase had 100 steps. Could two such staircases still form a rectangle? Use this to find the area of *one* staircase with 100 steps.

e. Find a rule for the sum of the first n integers:

$$1 + 2 + 3 + \cdots + n =$$

f. Connect this work back to your work on two numbers that share or don't share a common factor.

8. NO DEAL!

So did you remember?

Neat Stuff

9. For each coin flip data set below, we're giving you the total number of "runs" in the data. A "run" is any consecutive group of flips of the same type (heads or tails). For example, this data set has four runs:

That's right, folks, we're giving you the runs.

110100000

Which two data sets are real and which are fake?

a. This data set has 81 "runs" of heads or tails.

0110001010	0101000010	1011101010
0101001010	0101010010	0001010101
0111111010	1011010101	0101001110
1100100111	1110101010	0010000010

b. This data set has 60 "runs" of heads or tails.

1111100111	1010111011	1011010000
1101010010	1001010000	1011000101
0101100001	1010111011	0100000011
0000111111	1100000000	0101001110

c. This data set has 48 "runs" of heads or tails.

1101001111 0001110111 1111000011
1010001111 0001100100 0011101110
1101001111 1111001100 0000001000
1101110111 1101001111 0001001000

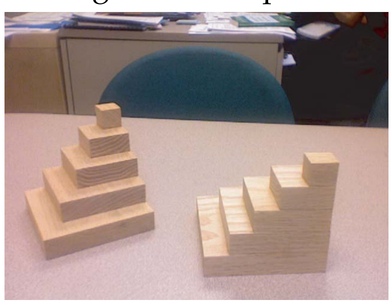
d. This data set has 65 "runs" of heads or tails.

0000101000 1101101001 0100001000
1110001010 1011011010 0010000100
0100001111 0011000011 1110111010
0110101000 1010100000 1011101000

10. How does the number of "runs" compare to the number of "switches" in a data set?

A "switch" is a spot where 01 or 10 is found in the data set, or perhaps a vehicle for Ellen Barkin and Jimmy Smits.

11. Here's a picture of two blocks of wood. Each is a 3-dimensional version of the staircase seen in Problem 7. Assume the side length of the topmost cube is 1 inch.



a. Determine the volume of each block of wood.

b. Show that the volume can also be calculated as a sum of squares of integers.

c. Show how six blocks in this shape can be fit together to form a solid box (or, better yet, build it). Then, find the dimensions of the box.

d. Describe how this process could be generalized to show that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Tough Stuff

12. *Courtesy ARML 2013.* For a positive integer n, let $C(n)$ equal the number of pairs of consecutive 1's in the binary representation of n. For example, $C(183) = C(10110111_2) = 3$. Compute $C(1) + C(2) + C(3) + \cdots + C(256)$.