# Problem Set 4: Anti-Match Game

## Opener

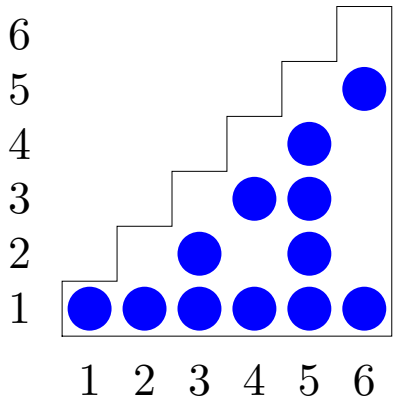
In today's game, players picked a positive integer up to 100, and won if the two numbers had no common factors greater than one. You're paired with Emo Phillips, who just picks randomly.

1. What is the probability of winning with Emo . . .
   - **a.** if your number is 2?
   - **b.** if your number is 5?
   - **c.** if your number is 10?
   - **d.** if your number is 20?
   - **e.** if your number is 25?

"Winning With Emo" was coincidentally one of the early challenges on *America's Next Top Goth*.

## Important Stuff

2. Here is a "staircase" with six stairs:




We've added dots wherever the two numbers have no common factor greater than 1.

Complete this table with the number of dots, and the proportion of the staircase that is covered in dots.

Kickin' in the front seat . . .
Sittin' in the back seat . . .
Gotta make my mind up . . . Which seat can I take?
(Sorry, but it is Friday.)

No table, huh? Looks like our work here is done.

| Stairs | Number of Dots | Staircase Area | Proportion |
|---|---|---|---|
| 1 | 1 | 1 | 1.000 |
| 2 | 2 | 3 | 0.667 |
| 3 | | | |
| 4 | | | |
| 5 | | 15 | |
| 6 | 12 | | 0.571 |
| 7 | | | |
| 8 | | | |
| 9 | | 45 | |
| 10 | | | |
| 15 | | | |
| 25 | 200 | | |
| 50 | 774 | | |
| 100 | 3044 | | |

Ohh. Never mind.

**3.** As $n$ gets larger, what appears to be happening with the proportion of the staircase that is covered in dots?

As of noon on July 3, we received 140 data sets at `http://bit.ly/fakeflips`. Here's a table summarizing the number of "runs" in all of the data we've collected so far.

As a reminder, the following coin flip sequence has 5 runs: 010011100.

| | real | fake |
|---|---|---|
| # runs $> 70$ | 4 | 19 |
| $50 \leqslant$ # runs $\leqslant 70$ | 66 | 47 |
| # runs $< 50$ | 0 | 4 |

**4.** **a.** Kelli will choose one of these data sets at random. What is the probability that her set is real? Fake?

**b.** Karen will choose one of these data sets at random, but only among those with more than 70 runs. What is the probability that her set is real? Fake?

**c.** Kemarie will choose one of these data sets at random, but only among those with 50 or more runs. What is the probability that her set is real? Fake?

**d.** Maryam will choose one of these data sets at random, but only among those with less than 50 runs. What is the probability that her set is real? Fake?

*FAKE!*

**5.** Millie hands you a new set of coin flips, not one of the original 140. She tells you that it has less than 50 runs. What might you be able to say about this new set of coin flips?

Careful, *original 140* also refers to the first-ever Tweet: "just setting up my twttr". But who was following??

**6.** Shelley is about to flip a fair coin four times, hoping to hit four heads in a row.

**a.** What is the probability that Shelley's first three flips are all heads?

**b.** What is the probability that Shelley's first four flips are all heads?

**c.** Suppose Shelley's first three flips are all heads. What is the probability that her fourth flip will be heads?

**d.** Why is the answer to part (c) different from the answer to part (b)? Be specific, and check in with others around you.

**7.** Steven is about to flip a fair coin four times, hoping to hit this exact sequence: heads, tails, tails, heads.

Maybe he just wants to write 1001 because it's so divisible!

**a.** What is the probability that Steven flips the exact sequence?

**b.** Suppose Steven's first two flips are heads then tails. What is the probability that his last two flips complete the sequence?

**c.** Suppose Steven's first two flips are heads then tails. What is the probability that his *middle two* flips are both tails?

**d.** Why is the answer to part (c) different from the answer to part (b)?

**8.** We want to read your flippin' tests! Please type in your test here:

`http://bit.ly/fliptestform`

Your test must be clear and specific: no guesswork involved! We promise to do something* with the submitted tests.

* Offer valid until 7/4/2013.

## Neat Stuff

**9.** **a.** In the opener, what one number can you pick to declare the game one, uh, I mean, won?

**b.** What is the *worst* possible number you can choose, and what is the probability of winning with that number?

**10.** Find the probability that, when flipping a coin 10 times, you never throw tails on two consecutive flips. Neat?

**11.** Tomorrow is 7/6/13, and 7 + 6 = 13. This will be the 7th time this has happened this year, and the time this has happened in Century 21. After 7/6/13 it will happen more times in the century, for a grand total of times. Cool!

Ralph said, "Last night I dreamed I was in a storm at sea. When I woke up, I was in the ."

**12.** Fourteen different people randomly select a positive integer up to 100. What is the probability that at least two of them pick the same number?

**13.** In the carnival game "Pile 'o Pennies", you can pick up and toss as many pennies as you like, all at once. If exactly four of them land heads, you win: no more, no less. What is the best choice for the number of coins to toss when playing this game?

Compute the probability that on the 4th, a firework landed in Bowen and Darryl's occupied hot tub. Oh wait, it's 1.

**14.** Build a histogram showing the probability of flipping exactly four heads on $n$ tosses.

**15.** Imagine the game from the opener, but now Emo picks a random positive integer up to 1 billion. You win if you match a common factor greater than one. Find a number that gives you the best possible chance of winning, and determine the probability of winning the game.

## Tough Stuff

**16.** Fourteen different people randomly select a positive integer up to 100. What is the probability that at least *three* of them pick the same number?

**17.** Here's a variant to the opener. Two people pick positive integers at random, with no limit. The numbers are revealed and the players are paid the value of the numbers' greatest common factor. It looks like this will frequently be \$1, but what is the expected payout of this game?

Infinite choice and randomization, oh my!