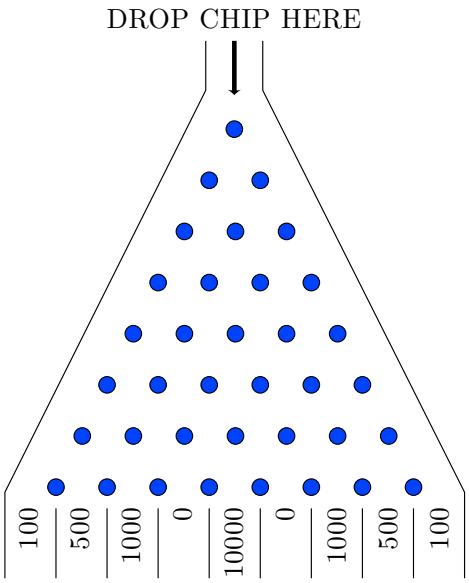
# Problem Set 6: Plinko de Mayo

## Opener

In Plinko, the player drops a chip and wins cash depending on where it lands. Here's a graphic of a simplified version of the Plinko board:

People get freaked out over Plinko. It's kind of nutty. Once on TPIR they had special guest Snoop Dogg, which is weird enough, and then he was super excited to help play Plinko.




Whenever the chip hits a peg, it can go left or right with equal probability. At the bottom, it will fall into one of nine slots with dollar amounts on them.

Good thing Peg's gone, or we'd be constantly hitting her with chips today.

1. Describe some ways in which our diagram is a simplified model of the real Plinko board, and how that might impact the accuracy of our model.

Model with aerobatics! No... um... Model with informatics! Er...

2. Use your work from Set 5 to determine the numbers in the 8th row of Pascal's Triangle.

3. Jacob is about to drop a chip. What is the probability that the chip will win $10,000?

4. Jill is about to drop a chip. What is the most likely thing that can happen, and how likely is it?

5. How much money, on average, is a Plinko chip worth?

At Phish concerts they sometimes have "Plinko jams", which are apparently post-modern in their self-referentiality, with a sound that is somehow both tacky and wet with an almost retro quality, as if the boys were playing an extra-terrestrial Fisher-Price pinball machine.

## Important Stuff

**6.** **a.** How many possible rearrangements are there for the letters LLRR?
**b.** Expand $(L + R)^4$ and compare.

LLRR comes from the planet Omicron Persei 8.

**7.** Monica challenges you to add up the numbers in the 6th row of Pascal's Triangle. The other Monica knows a trick to calculate it quickly. What's the sum? What's the trick, and why does it work?

**8.** **a.** Use the 8th row of Pascal's Triangle to write the results of expanding $(L + R)^8$. Use tech if needed.
**b.** Raziyeh notes that a Plinko chip must go left 4 times and right 4 times to land in the \$10,000 spot. How many different ways can this happen?

Possible technology includes the TI-Nspire CAS, websites like Wolfram Alpha, programs like Geogebra, and nerds.

**9.** The notation $\binom{8}{4} = 70$ refers to a specific spot in Pascal's Triangle, in the 8th row. It is also the number of rearrangements of LLLLRRRR, the number of ways to choose 4 lefts (and 4 rights) from 8 possible moves.
**a.** What is the value of $\binom{8}{0}$? What, in general, is the value of $\binom{n}{0}$? Why does this happen?
**b.** What is the value of $\binom{n}{1}$? Why?
**c.** Which is bigger, $\binom{7}{2}$ or $\binom{7}{5}$? Why?

**10.** This table gives the number of dots $d(n)$ in a staircase with $n$ steps. The third column marked with $\Delta$ contains the difference between consecutive values of $d(n)$.

| $n$ | $d(n)$ | $\Delta$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 4 | 2 |
| 4 | 6 | 4 |
| 5 | 10 | 2 |
| 6 | 12 | 6 |
| 7 | 18 | |
| 8 | 22 | |
| 9 | 28 | |

| $n$ | $d(n)$ | $\Delta$ |
|---|---|---|
| 10 | 32 | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | 72 | |
| 16 | | |
| 17 | | |
| 18 | | |

What did we learn from George Hart yesterday? I learned that if you cut Spongebob Squarepants along the main diagonal you get Patrick Star.

Plinko was first played on January 3, 1983, the exact same day that \$ale of the Century first aired. Truly a great day for game shows.

**11.** The staircase with 3,000 steps has 2,736,188 dots. Use this information to calculate the number and proportion of dots in a staircase with 3,001 steps.

It would sure be nice if 3001 happens to be a prime number. Woo hoo!

**12.** Get your own real and fake 120 coin flips from Set 1. Think of the 120 coin flips as 60 independent two-flip sequences. For example, if your first 8 coin flips are 01001011, then your first four two-flip sequences are 01, 00, 10, and 11.

- **a.** Start with your real flip data. Tally up the number of two-flip sequences that had no heads, one head, and two heads. (Your answers should add up to 60.)
- **b.** Repeat for your fake coin flip data.
- **c.** If you flip two fair coins, what is the probability that you get no heads? One head? Two heads?
- **d.** In 60 two-flip sequences, how many should have no heads? One head? Two heads?

If your answers add up to 119, go back to Problem 7d on Set 4.

**13.** Test sequences to determine whether these data sets are real or fake.

Or maybe these are instructions for dropping chips on giant Plinko boards . . . Plinko has to be the only TPIR game with its own Urban Dictionary page.

**a.**

| | | |
|---|---|---|
| 0010010110 | 0111000110 | 1001101011 |
| 0100011010 | 1010001000 | 0100000101 |
| 0000111010 | 1111010001 | 0000011010 |
| 1011110111 | 0000010011 | 1111111111 |

**b.**

| | | |
|---|---|---|
| 1101110001 | 1111010110 | 0000001011 |
| 1011010110 | 0011110010 | 0001101011 |
| 1000011000 | 1100001111 | 0011000100 |
| 1111011010 | 1100001011 | 1100111100 |

**c.**

| | | |
|---|---|---|
| 0111001011 | 1100010110 | 1010000111 |
| 0011011100 | 0000001001 | 1000011000 |
| 1010111010 | 0011010110 | 1110111111 |
| 1010010001 | 0010100001 | 1011101100 |

It may be interesting to test more than just two-flip sequences . . .

## Neat Stuff

**14.** 
- **a.** Find the values of $\binom{2}{2}$ through $\binom{10}{2}$ using Pascal's Triangle or any other means.
- **b.** What's up with these numbers? Can you find a connection or explanation?

There they are, I found 'em!

**15.** The staircase with 900 steps has 246,326 dots. Use this information to calculate the number and proportion of dots in a staircase with 901 steps.

It would sure be nice if 901 happens to be a prime number. Oops!

**16.** Esther pauses a Plinko game in progress, with the chip directly above the \$0 space, two rows up the board. The chip could fall into the \$1000, \$0, or \$10,000 spaces. At that exact moment, how much is the Plinko chip worth?

Note: this is different from a stuck chip, which leads to the getting of the trusty 'ol Plinko Stick and a do-over. No, seriously, that's what happens.

**17.** The actual Plinko board is 12 rows tall, has "walls", and the player can drop their chip from any of 9 different places (above any of the scoring slots). See what you can do to come up with the actual expected value of the game, or the actual probability of having a chip fall into the \$10,000 slot.

According to the Internet, Plinko covers all five of Aquinas's Proofs for God's existence.

**18.** Devise an experiment using one or more six-sided dice with exactly a one-*fifth* probability of success.

**19.** Here again is $F_5$, the *Farey sequence* of order 5:

They're in increasing order, with maximum denominator 5.

$$\left\{\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}\right\}$$

Determine $|F_{15}|$, the *number of elements* in $F_{15}$, by thinking about how you go from one Farey sequence to the next.

**20.** Suppose integers $a$ and $b$ are picked with $1 \leqslant a \leqslant b \leqslant 15$. What is the approximate probability that the fraction $\frac{a}{b}$ is written in lowest terms?

**21.** Set up a coordinate grid with $1 \leqslant y \leqslant x \leqslant 15$. Place a blue dot at the integer lattice point $(x, y)$ if the fraction $\frac{y}{x}$ is in lowest terms. Notice anything?

For more about Plinko, consult your local library, or read then completely reject the recent NPR article, "Let Us Now Have A Heated Argument About Plinko. I'll Start".

### Tough Stuff

**22.** Find a finite-length game based entirely on coin flipping that has probability of success $p = \frac{1}{3}$, or show that such a game is impossible.