

Problem Set 8: Spin the Wheel

Opener

Annemarie, Barb and Clint each spin the Big Wheel twice. The Wheel is marked with the numbers 25, 50, 75, and 100. Players earn a total combined value for the two spins.

1. What are the largest and smallest totals each person could earn from the two spins? How likely are these possibilities?
2. What is the most likely total each person could earn from the two spins, and how likely is it?
3. What's the result when you multiply out this expression:

$$(x^{25} + x^{50} + x^{75} + x^{100})^2$$

What up with that? I say, what up with that??

Boop boop boop boop
 boop boop boop boop
 boop boop boop boop
 boop boop boop boop boop
 boop boop boop boop .
 boop .. boop boop
 boop boooooop

 booooooop

Oooo weeee . . .

Important Stuff

4.
 - a. How many ways are there to rearrange the letters in HHHHHTTTT?
 - b. . . . HHHHTTTT?
 - c. How many total ways are there to flip 8 coins?
 - d. What's the probability of flipping 8 coins and getting exactly 4 heads and 4 tails?
 - e. What do you get when you multiply out $(H + T)^8$?
Don't do it by hand, please!

You could flip them on a boat . . . you could flip them while you vote.

5. Why does this expression represent the number of ways to rearrange the letters in HHHTTTT?

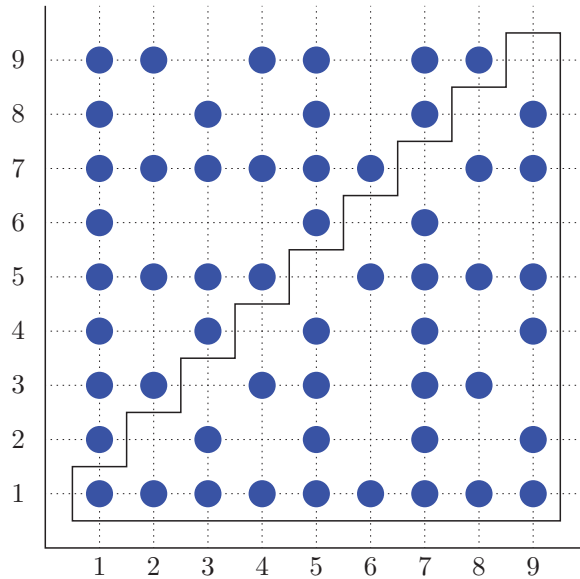
$$\frac{10!}{(3!)(7!)}$$

We don't care if you've memorized some fancy formula. Why? Why? Why????

6. What is the probability of flipping 120 coins and getting exactly 60 heads and 60 tails?
7. What is the probability of flipping 120 coins and getting a 61-59 split of some kind?

Compare your answer with what you wrote for Problem 7 on Set 1.

8. Pictured here is a 9-by-9 grid. At coordinates (a, b) is a blue dot if the greatest common factor of a and b is 1.



Actual dot color may vary. Unlike Tobias, these dots may not just blue themselves.

- k. How many blue dots are in the 9-by-9 staircase in the bottom right? Chances are good that you already have this answer, but you can count 'em again if you'd rather.
 - y. How many blue dots, in total, are in the 9-by-9 grid?
 - l. How can you use the number of dots in the staircase to tell you how many dots there are in the grid?
 - e. What proportion of the overall 9-by-9 grid is covered in dots?
9. Build a grid for $1 \leq x \leq 12$ and $1 \leq y \leq 12$, plotting (a, b) if the greatest common factor of a and b is 1.
- a. How many points did you plot?
 - b. What proportion of the 144 points in the grid did you plot?
10. Complete the table on the next page with the number of dots in each staircase and the corresponding grid, along with the proportion of each grid that is plotted.
11. As grids get larger, what appears to happen to the proportion of the grid that is covered in dots?

You could count them in a notebook . . . you could count them just by rote.

You could plot them in a coat . . . you could plot them and then gloat.

| Size | Staircase Dots | Grid Dots | Grid Area | Proportion |
|-------|----------------|-----------|-----------|------------|
| 1 | 1 | 1 | 1 | 1.000 |
| 2 | 2 | 3 | 4 | 0.750 |
| 3 | 4 | | 9 | |
| 4 | | | 16 | |
| 5 | | | 25 | |
| 6 | | 23 | | |
| 7 | | | | |
| 8 | | | | |
| 9 | 28 | | 81 | |
| 10 | | | | |
| 15 | | | | |
| 25 | 200 | | | |
| 50 | 774 | | | |
| 100 | 3,044 | | | |
| 500 | 76,116 | | | |
| 3,000 | 2,736,188 | | | |

The 7-11 corporation has announced that free Slurpees will only be handed out if the numbers 7 and 11 appear consecutively in a table on today's problem set.

Also, if you have taken a napkin from the lunch area, and your name happens to be Nadine . . . hi.

Neat Stuff

12. You typed your 120 coin flips in groups of 10.
 - a. What's the probability that a group of 10 real coin flips has exactly 5 heads and 5 tails?
 - b. Exactly 4 heads? 6 heads? 3 heads? 7 heads?
 - c. Abdullah calls a group of 10 flips "*extreme!*" if it has at least 8 of one type (heads or tails). What is the probability that a group of 10 real flips is *extreme!*?
13. A set of 120 real flips makes 12 groups of 10 flips each.
 - a. What is the expected number of the 12 groups that have exactly 5 heads and 5 tails?
 - b. What is the expected number of groups that have exactly 4 heads? 6? 3? 7?
 - c. What is the expected number of *extreme!* groups?
14.
 - a. What is the probability that *none* of the 12 groups of 10 coin flips is *extreme!*?
 - b. What is the probability that *none* of the 12 groups have exactly 5 heads and 5 tails?

You could flip them near a moat . . . "you could flip them", and I quote.

We're still keeping it real in these two problems.

15. Alex asks you to explain why the center number(s) in a row of Pascal's Triangle is largest. There might be more than one good explanation!
16. Find some connections between trains, coin flips, and Pascal's Traingle. Hee hee.
17. Using only red (2) Cuisenaire rods, how many ways can you fill a 2-by-10 rectangle?
18. In the dice game *Farkle*, a player rolls a bunch of dice and farkles (a.k.a. loses) if none of the dice show a 1 or 5. What is the probability of farkling if you roll one die? 2? 3? 4? 5? 6?
19. Today, each 7-11 store manager independently decides which four Slurpee flavors to feature, out of a menu of 24 flavors. Joseph is willing to go to all four of the 7-11 stores near Park City to hunt down his favorite flavor.
 - a. What is the probability that Joseph finds his flavor at the first store?
 - b. What is the probability that Joseph will get turned away by all four stores, one after another?
20. The real Big Wheel has numbers from 5 to 100 in steps of 5 and the highest player total wins *as long as it's 100 or less*. As the last spinner your "score to beat" is 70. What's your probability of winning?

Please include in your explanation the reasons why Oklahoma 7-11s don't sell Slurpees.

Farkling. Farkling. Farkling. It's fun to say!

Joseph's favorite Slurpee is vanilla chai cherry amazing technicolor Dr. Pepper.

Ties are possible and result in a "spin-off". No, not like *Joey* or *The Jeffersons*.

Tough Stuff

21. When spinning the Big Wheel, a contestant may choose to stop after one spin, or keep going to improve their total (and risk going over 100).
 - a. If you're the *second* contestant to spin, what's the best possible strategy and how likely is it to win? Assume the first contestant already busted out.
 - b. If you're the *first* contestant to spin, what's the best possible strategy and how likely is it to win?

You could spin it with strep throat . . . you could spin it then be smote.

You could drink a root beer float. You could boop a quarter note. You could sow a wild oat. You could run out of space and ideas while writing this.