Problem Set 10: Two-Bedroom Coin-Die-Minimum

Opener

Blue and Red play a game. Blue has some number of coins (from 2 to 15), and Red has a standard six-sided die. Red rolls the die to set a target number. Blue then tosses all his coins. Blue wins if he tosses *more* heads than the target number. Red wins if Blue doesn't beat the target number.

Better than cookies, right?

1. In your group you've been given k, a specific number of coins for Blue. Compute the probability that Blue wins this game *given that* Red rolled a 1.

2. Compute the probability that Blue wins for each possible Red roll. It's okay to approximate the probabilities to four decimal places.

It's safe. It's very safe.

3. Compute the probability that Blue wins for the specific number of coins you were given. Reach concensus in your group and compile your group's information here:

`http://bit.ly/bluevredflips`

4. Start over, but this time Blue has $17-k$ coins, where k is the number of coins you used for Blue in Problem 1.

I was told there was to be no algebra before 9 am. But here there is algebra, so I'm not happy.

Important Stuff

5. Thea and Trevor repeatedly play a game. It appears to be fair, but actually the game is rigged in favor of Thea: she wins with probability 0.7.

Shh, don't tell Trevor. Oops, too late.

- **a.** If they play the game twice, how likely is it for Thea to win twice? For each to win once? For Trevor to win twice?
- **b.** If they play the game three times, what could happen? Use a tree diagram if you like.
- **c.** If they play the game four times, how likely is it for them to each win twice?

Oops, they played it again . . .

6.
- **a.** Expand $(0.7x+0.3y)^2$ any way you like and discuss.
- **b.** Expand $(0.7x + 0.3y)^3$ and discuss some more.
- **c.** What is the coefficient of x^2y^2 in the expansion of $(0.7x + 0.3y)^4$?

7. Thea and Trevor continue to play their game. If they play 10 times, find the probability that Thea wins more than 5 games out of 10. (Use a polynomial expansion!)

This does not count as a polynomial expansion: $x^2 + 5x + 6$

8. For 10 plays of Thea and Trevor's game, build a histogram with the number of wins for Thea on the horizontal and the probability of achieving that many wins on the vertical. Describe the shape of the histogram!!

9. Use a polynomial expansion to find the probability that Blue tosses at least 7 heads on 10 coin flips.

10. For 10 fair coin flips, draw a histogram with the number of heads on the horizontal and the probability of flipping that many heads on the vertical.

11. Titin says that the polynomial expansion works because "events are independent." What does she mean by that? Is this reasonable?

12. A woman in her 40s has roughly a 1% chance of having breast cancer. Many women get mammograms to test for breast cancer: the mammogram test gives the correct answer 90% of the time and an incorrect answer 10% of the time. What is the probability that a woman has breast cancer, given that the mammogram came back "positive"?

This problem may be easier if you pick a count for the number of women being tested. Just don't pick Count Von Count, he will just keep yelling numbers at you.

Neat Stuff

13. What is the probability that a woman has breast cancer, given that two mammograms have come back positive?

14. If you pick *two* positive integers at random, what is the probability that their greatest common factor is 2?

Two! Two positive integers, ah ha ha.

15. Here's a sequence of 11 coin flips with 5 "runs":

HHH TT H TTTT H

Eleven! Eleven coin flips, ah ha ha.

a. There is another sequence of 11 coin flips with 5 runs where the runs are in exactly the same places. Write down that sequence.

b. This 11-flip sequence can also be thought of as a 10-*instruction* sequence where the instructions are "run" and "switch". For the sequence above the instruction sequence is

RRSRSSRRRS

Ten! Ten instructions, ah ha ha.

Write down several 11-flip sequences and their corresponding 10-instruction sequences until you get the hang of it.

c. If you pick an 11-flip sequence at random, what is the probability that it has exactly 5 runs?

Five! Five ru . . . shut up, Count!

16. Build a histogram for the distribution of the number of runs when flipping a fair coin 11 times. On the horizontal is the number of runs (from 1 to 11) and on the vertical is the probability of getting that many runs. Interesting!

17. **a.** Take your real and fake 120-coin-flip data and build two histograms. On the horizontal is the length of runs in your data, from 1 to ... more than 1. On the vertical is the number of runs of that specific length.

For the flips in problem 15, there are two runs of length 1, one of length 2, one of length 3, and one of length 4.

b. How do the histograms compare?

c. What is the expected number of runs of length 1 in 120 coin flips? What is the expected number of runs of length 2?

18. What is the probability that you will answer this problem correctly?

19. Mary Ann and Mimi play a "best-of-7" series of Don't Spill The Beans. Mimi has a $p = 0.6$ probability of winning each game. What is the probability that Mimi wins the series by winning 4 games before Mary Ann does?

This is not the only thing you should try not to do with beans.

20. Joanne and Jonathan play a game of Pick a Card. Jonathan is cheating and has a $p = \frac{2}{3}$ probability of earning a point on a turn, while Joanne only has a $\frac{1}{3}$ probability. The first player to 10 points wins. What is the probability that Joanne wins the game unfair and square?

The stakes are high: Joanne could lose as much as 100 million yen!

21. **a.** When flipping a fair coin 120 times, how likely it is to get exactly 60 runs (strings of consecutive heads or tails)? 59? 58? 61? You may want to use the ideas from problem 15. Techmology is your friend!

b. One group proposed this test: a set of 120 coin flips with less than 50 or more than 70 runs is fake. What is the probability that a set of 120 real coin flips will have between 50 and 70 runs (inclusive)? Compare your answer to the data from Problem 3 on Set 4.

Highly recommended is the Ali G routine about techmology. He asks if there will ever be a computer that can mutliply nine nine nine nine . . . nine nine eight nine point nine nine . . . nine times nine nine . . .

22. Suppose you are given a data set: you're not told if it's real or fake. The data set does *not* have between 50 and 70 runs. How likely is it that the data set is fake?

23. **a.** When rolling two standard dice, what totals are possible and how likely is each total?

b. Suppose one die has its six faces numbered 1,1,3,3,5,5 and other numbered 2,2,4,4,6,6. What totals are possible now and how likely is each total?

c. Find a way to renumber the sides of two dice so that the new dice have the same outcomes and probabilities as two standard dice.

d. Find a way to renumber the sides of two dice *with positive integers* so that the new dice have the same outcomes and probabilities as two standard dice, or show that it is impossible to do so.

It's still allowed to repeat a number on a die, or to use larger or smaller numbers than 1-6.

Tough Stuff

24. **a.** Determine the probability that on 10 consecutive coin flips, you never flip heads twice in a row.

b. Determine the probability that on 11 consecutive coin flips, the longest "run" is exactly 2.

c. Determine the probability that on 11 consecutive coin flips, the longest "run" is exactly 3.

25. Find the probability that three positive integers chosen at random do not *all* share a common factor greater than 1.

For example, the set $\{15, 21, 35\}$ do not *all* share a common factor.

26. Read the 14 proofs at this website:

`http://www.secamlocal.ex.ac.uk/people/staff/rjchapma/etc/zeta2.pdf`

Which one do you like best? We hope at least one of them amazes you!