Day 1: Party in Triangle USA

Welcome to PCMI! We know you'll learn a great deal of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How do others think about this question?
- **Be excellent to each other.** Believe that you have something to learn from everyone else. Remember that everyone works at a different pace. Give everyone equal opportunity to express themselves.
- **Teach only if you have to.** You may feel the temptation to teach others in your group. Fight it! We don't mean you should ignore your classmates but give everyone the chance to discover. If you think it's a good time to teach your colleagues about the Law of Cosines, think again: the problems should lead to the appropriate mathematics rather than requiring it.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff. Check out the Opener and the Important Stuff first. All the mathematics that is central to the course can be found and developed there. *That's* why it's Important Stuff. Everything else is just neat or tough. Each Day is based on what happened in the previous set, and what happened in the previous *class*.

PCMI teachers have solved two previously unsolved problems presented in these courses.

When you get to Day 3, come back and read this again.

Will you remember? Maybe . . .

Opener

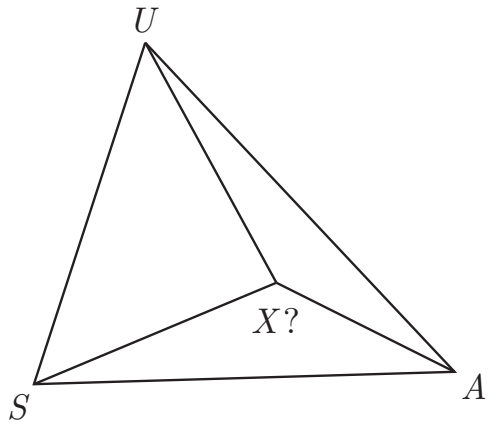
1. **a.** If you have a computer or tablet, get GeoGebra on it. Go to `http://www.geogebra.org`, then click on "Download now". Once you've got it working, skip to Problem 2.

GeoGebra is clearly not from Portland, since it doesn't have a bird on it. Bah this joke sucks.

b. *Work on this only when you have returned from break!*

Miley wants to have a party in triangle USA. Where should the party be located to minimize the total distance that people from U, S, and A must travel?

So I put my hands up, they're yodelin' my song . . .



The goal is to find X so that $UX + SX + AX$ is as small as possible.

Create this diagram, then figure out where point X should be placed. Is there anything special about that point?

For today, work on the GeoGebra sketch in teams of two or three, consisting of the people on one side of the table. Once you've got the sketch working, each person can go back and recreate the sketch on her/his own computer if desired, but start off working as a group.

Important Stuff

2. Julia's rectangle has perimeter 12 and area 9. Find its dimensions.

3. Nancy tells you the perimeter and area of a rectangle. Is it always possible to confidently determine the dimensions of the rectangle?

4. Here are some perimeters and areas. Find the dimensions of each rectangle.

a. perimeter 24, area 36
b. perimeter 24, area 35
c. perimeter 24, area 32
d. perimeter 24, area 27
e. perimeter 24, area 34
f. perimeter 24, area 37

5. Find the values of x that make each equation true.

Hey I found them, they were hiding on the other page!

a. $x^2 - 12x + 36 = 0$
b. $x^2 - 12x + 35 = 0$
c. $x^2 - 12x + 32 = 0$
d. $x^2 - 12x + 27 = 0$
e. $x^2 - 12x + 34 = 0$
f. $x^2 - 12x + 37 = 0$

6. A rectangle has perimeter 36. What could its area be?

36 inches? 36 yards? 36 furlongs? 36 smoots? 36 football fields? 36 beard-seconds? Please forgive us for our generally sloppy use of non-units throughout the course. Also, beard-seconds are awesome.

7. Sheana's rectangular box has surface area 96 and volume 64. Find its dimensions.

8. Phiola tells you the surface area and volume of a rectangular box. Is it always possible to confidently determine the dimensions of the box?

Neat Stuff

9. For each point, decide if it is the same distance from $(7, 1)$ and $(-2, 9)$.

Hm, this looks like multiple choice, but NO!

a. $(7, 10)$
b. $(-1, 1)$
c. $(-17, -17)$
d. $(-2, 0)$

10. Find some rectangles whose perimeter and area have the same numeric value. Try to find them all. No, there's more.

11. Find some rectangular boxes whose surface area and volume have the same numeric value. How many are there?

12. Patricia's triangle has perimeter $6\sqrt{3}$ and area $3\sqrt{3}$. Find its dimensions.

Now it's time for Annie's secret message.

13. Chris tells you the perimeter and area of a triangle. Is it possible to confidently determine the side lengths of the triangle?

2-5-19-21-18-5-20-15-4-18-9-14-11-25-15-21-18-23-1-20-5-18

14. Find some triangles whose perimeter and area have the same numeric value.

Tough Stuff

15. A triangle has perimeter 24. Find its maximum possible area, and explain how you know that this *must* be it.

16. Given positive integer n, the unit fraction $\frac{1}{n}$ can be written as the sum of two other unit fractions:

$$\frac{1}{n} = \frac{1}{a} + \frac{1}{b}$$

Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of two unit fractions.

Like the blood type, a and b must be positive. Unlike the blood type, they must be integers.

17. Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of *three* unit fractions.

18. Some triangles are acute, some are right, and some are obtuse. What fraction of triangles are acute? What fraction are right? What fraction are obtuse?

Bonus: determine the sum of these three fractions. Wait a minute . . .