Broadening Teacher Experiences in Preparation for Teaching Statistics

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Introduction

In 2007 the writers of the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) *Report* state, "Every high-school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizen, employment, and family and to be prepared for healthy, happy, and productive life" (Franklin, et al., 2007) Many teachers understand the importance of teaching statistical topics but may have limited experiences teaching those topics. In addition, because the study of statistics requires students to grapple with uncertainty and variability in data, the teaching of statistics may require a shift in pedagogical methods that engage students in problems that are exploratory and open-ended. In 2015, the American Statistical Association (ASA) commissioned the writing of *Statistics Education of Teachers* (SET) to emphasize teachers' statistical preparation (Franklin, et al., 2015). The SET report recommends that programs for teachers develop pedagogical content knowledge necessary for effective teaching of statistics. This knowledge includes being familiar with common student conceptions, content-specific teaching strategies, strategies for assessing statistical knowledge, and appropriate integration of technology for developing statistical concepts.

In this brief, we provide information and ideas to support teachers whose goals include expanding the opportunities for their students to engage in rich statistical tasks within the broader mathematics curriculum. The main areas of teacher support considered in this brief are:

Building a Collaborative Teacher Community,
Preparing and Implementing Statistical Tasks, and
Assessing Student Progress through Statistical Tasks.

We recognize there are differences between experiences that can support pre-service teachers and those that can support in-service teachers. In some countries, in-service teachers have very limited time to address the three main areas in this document, but we offer some suggestions that may support all teachers in building community, preparing and implementing statistical tasks, and assessing student progress through statistical tasks.

Building a Collaborative Teacher Community

In many countries, teachers do not have the dedicated time to share information with each other about how to develop or sequence the topics of a course. As a consequence, classes are developed according to the discretion of each teacher and might not be based on needs or basic skills that students in each grade level should have. In the case of statistics, many teachers have not had the opportunity to learn the concepts or consider the pedagogical methods for the effective teaching of statistical topics.

We believe that teachers need a way to communicate with each other in order to exchange information and experiences in the hopes of improving their practice. Because of the limited time during the working school day and the lack of teacher professional development time, we

see the need to develop a virtual space that teachers can easily access and share information regarding their experiences, curricular content/lessons, assignments and tutorials with the aim of improving the educational process.

Development: We are calling for the development and support of a virtual community that will offer an opportunity for teachers to develop a sense of community and exchange of knowledge specific to the teaching and learning of statistics.

A possible suggestion is to create a website for both in-service and pre-service teachers.

Access. Free

Content. The website will offer various resources such as:

- Links to pages with broad academic content, in order to reinforce concepts.
- Links to a list of existing experiments and projects classified according to statistical concepts.
- Space for teachers to contribute to their own projects and activities as they develop them.
- Links to statistical tools so teachers can access software downloads or applications that can be run from websites.
- Tutorial videos on how to use various technological tools to enhance the teaching and learning of statistics.
- A platform where teachers can pose their own questions and answer the questions posed by their virtual colleagues.
- Chat rooms where teachers can have the opportunity to communicate with other teachers concerning the teaching of statistics.

Feasibility. Since it is a virtual community for teachers from different countries, we recognize that it may be difficult to ensure the stability and effectiveness of the website. We hope that people/industries in a position to provide such a website/service will consider this request and the potential impact of such a virtual community on the statistical education of students across the globe.

Responsibility: After the initial creation of the website, each of the participating countries will be responsible for increasing the items of interest in the site.

A possible list of resources for the website include:

- GAISE Report in English and Spanish - http://www.amstat.org/education/gaise/
- *NCTM's Principles to Actions* in English and Spanish www.nctm.org/PtA/.
- *Five Practices for Orchestrating Productive Mathematical Discussions* in English and Spanish www.nctm.org/store/Products/(eBook)-5-Practices-for-Orchestrating-Productive-Mathematics-Discussions-(PDF)/.
- American Statistical Association http://www.amstat.org/
  Resources for Teachers SET www.amstat.org/education/SET/SET.pdf

and the STatistics Education Web (STEW) www.amstat.org/education/stew/

- Websites with Simulation and Data Analysis Tools:
  NCTM Core Tools www.nctm.org/corematthtools/
  Texas Instruments Building Concepts Statistics and Probability
  www.tibuildingconcepts.com/activities/statistics

Preparing and Implementing Statistical Tasks

When we consider broadening the experiences of teachers as they prepare to teach statistical topics, we consider one of the National Council of Teachers of Mathematics's (NCTM) Guiding Principles from *Principles to Actions*:

> Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically (NCTM, 2014, p. 5).

In the area of statistics, some of the key elements of reasoning and sense-making are analyzing data, drawing appropriate conclusions from data and interpreting results. In this brief, we will share an example to illustrate how teachers can engage their students in reasoning and sense-making specific to topics in statistics.

We address some broad pedagogical considerations and connect those to this specific example focused on statistical topics. The broad pedagogical ideas that we would like to focus on are the *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011).

Anticipating what students will do--what strategies they will use--in solving a problem;
Monitoring their work as they approach the problem in class;
Selecting students whose strategies are worth discussing in class;
Sequencing those students' presentations to maximize their potential to increase students' learning;
Connecting the strategies and ideas in a way that helps students understand the mathematics learned.

Example - Fútbol (Soccer): A Statistical Investigation

The following example was adapted from an activity published in a report written by two preservice teachers who carried out their first teaching practices in the 4th year of an Argentinean secondary school (14-15 years old). The whole report is available at: http://www2.famaf.unc.edu.ar/institucional/biblioteca/trabajos/6085/17150.pdf (Nanini &Sierra, 2014).

The school practice lasted four weeks, and we will concentrate on the introductory activity. The main goal was to allow the students without any theoretical knowledge related to analysis of statistical data to analyze tables and histograms of data related to fútbol (soccer). The task was prepared in 2014 just before the FIFA World Cup in Brazil was finished. The lesson assumed that students would have access to a spreadsheet tool.

The fútbol players included in the activity are Lionel Messi, an Argentinian who plays in Spain, and Cristiano Ronaldo, a Portuguese who also plays in Spain. By the time of the implementation of the activity, Messi was playing for FC Barcelona and Ronaldo for Real Madrid Club.

The table in Figure 1 shows some statistics about Messi's performance between 2004 and 2014 while playing for FC Barcelona.

SEASONS	MESSI'S MATCHES PLAYED	GOALS	MEAN GOAL
2004/05	9	1	0.11
2005/06	25	8	0.32
2006/07	36	17	0.47
2007/08	40	16	0.40
2008/09	51	38	0.75
2009/10	53	47	0.89
2010/11	55	53	0.96
2011/12	60	73	1.22
2012/13	50	60	1.20
2013/14	46	41	0.89

Figure 1. Messi's statistics

1. What are the variables and types of variables provided in the table?
2. Where do you see the number of goals that Messi scored in a given season?
3. How can you analyze whether the number of games Messi played was always the same or if there were variations? In what season did he play the most matches?
4. In which season did he score the most goals and in which season did he score the least number of goals?
5. Why is it that in the 2004/2005 season he only scored one goal? Can there be another factor that does not appear in the table?
6. Explain what information is given in the column to the far right. How do you think these values were calculated?
7. In which season did he have the highest scoring average? Make a conjecture about what might have happened in that season.

A second table presenting Ronaldo's performance over the same seasons as Messi's was given to the students (see Figure 2) and new questions were posed.

SEASONS	RONALDO'S MATCHES PLAYED	GOALS	MEAN GOAL
2004/05	50	9	0.18
2005/06	47	12	0.26
2006/07	53	23	0.43
2007/08	49	42	0.86
2008/09	53	26	0.49
2009/10	35	33	0.94
2010/11	54	53	0.98
2011/12	55	60	1.09
2012/13	55	55	1.00
2013/14	47	51	1.09

Figure 2. Ronaldo's statistics

8. Analyze and compare the data from the 2010/11 season for both players.
9. Which player has the highest scoring average?
10. Which season was the best for each player? Why?

A comparative table merging data from both soccer players (Figure 3) and a graphical representation of such data (Figure 4) are presented:

SEASONS	MESSI'S GOALS	RONALDO'S GOALS
2004/05	1	9
2005/06	8	12
2006/07	17	23
2007/08	16	42
2008/09	38	26
2009/10	47	33
2010/11	53	53
2011/12	73	60
2012/13	60	55
2013/14	41	51

Figure 3. A comparative table.

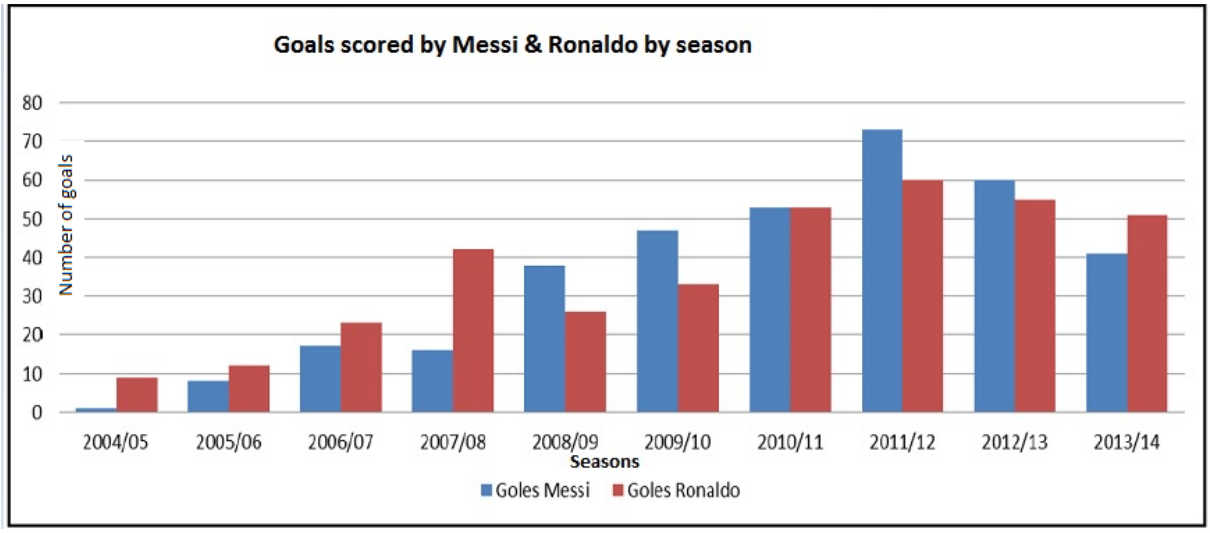

Figure 4. A comparative graph.

11. In what season did Messi and Ronaldo score the same number of goals?
12. Is there a period of seasons that Messi scored more goals than Ronaldo?
13. Is there a period of seasons when the number of Ronaldo's goals decreased?
14. Are there advantages in looking at the graph instead of the table? If so, what are those advantages?

You may want to provide other questions that encourage students to create new graphical representations of the given data. For example, students could create a dot chart to compare the goal averages for each player over time or a chart with time along the horizontal axis and the goal averages for each player along the vertical axis (called a line graph).

The last task (#15) puts the students in the situation of having to make a justified decision about which of the two soccer players should be recognized as the best soccer player of the last ten years:

15. Suppose that FIFA wants to award a prize to the player who has been the "best" over the last 10 years. Using five pairs of data from the given table or the given graph, write a brief explanation that argues which of the players should receive the prize.

Each one of the 15 questions were associated with different goals such as:

- Read and interpret data in a table considering the data in each column and the relationships among the data in each column.
- Distinguish among different types of variables and their maximum and minimum values.
- Recognize that data/information not shown in a table or a graph may affect drawing conclusions from the data presented.
- Introduce the idea of mean related to a particular context.
- Compare data, and recognize the advantages and disadvantages of different ways of representing them.
- Make decisions and communicate the reasons for those decisions based on the data analysis.
- Create other graphical representations for the data

Connection to Five Practices
To illustrate each of the five practices for orchestrating productive mathematics discussions (Smith & Stein, 2011), we offer some suggestions specific to the Example - Fútbol (Soccer): A Statistical Investigation.

Anticipating *what students will do--what strategies they will use--in solving a problem*
In anticipating students' strategies and answers, teachers might consider whether the context is familiar to all the students, possible difficulties that students may have or the specific statistical concepts that students will call upon as they analyze the given data. Initially, when students have to make a decision about the fútbol player who deserves to win the prize, they may answer based on their personal preferences and not on the provided data. According to the questions posed in the task, it might be useful to have access to extra information in order to better know the context from which the data comes and to be aware that sometimes it is not possible to answer a question with the information available in the tables or the graphs. The notion of mean will arise during the activity, through the variable "goal mean". The idea of average may arise leading to the need for a discussion on how to calculate a mean.

Monitoring *student work as they approach the problem in class*
As students work in groups on the project, the teacher should circulate about the room, noting the work of each group and making note of the students' questions and conceptions. There may be times when the teacher initiates a whole-class discussion so that students can share their thinking on how to read graphs and tables. After the teacher has a global view of student work, he/she may ask probing questions to reveal other information about students' understanding of the data. For example: How are the students reading the tables? Some students might focus on only one aspect of the data comparing a row of data for each of the players in a particular season. Some students might focus on only one player and look at the data for that player over an extended period of time.

Selecting *students whose strategies are worth discussing in class*
In selecting students' strategies, it may be helpful to choose varying strategies as noted in *Monitoring*. Teachers might choose students who demonstrated a confusion with mean and percentages in trying to understand how to calculate the rightmost column of values. Some students might create their own graphical representation for the data. For example, they might choose to create a dot plot of the goal averages for each player over time. With regard to choosing the "best" player, some students might choose Messi and others might choose Ronaldo. As teachers observe the students' reasoning in each case, they might select one student from each "camp" to share their work later during the class.

Sequencing *the students' presentations to maximize their potential to increase student learning*
Before considering the sequencing of student presentations it is important to be sure that there has been careful attention placed on creating an environment where students feel "safe" sharing their ideas and understand that it is important to learn from our mistakes. In sequencing students' presentations, teacher should consider the varying levels of difficulties that students may have analyzing and representing the data and thus the level of complexity in the varying solutions. Perhaps the teacher will choose to sequence the sharing of those

solutions from less complex to more complex or group those that focus on numerical strategies to make a decision as opposed to those using graphical representations.

Connecting *the strategies and ideas discussed in a way that helps students understand the mathematics learned*

The use of graphical representations helps students visualize the information contained in the data. Although some students might note that the tables seem to offer "more accurate data" than the graph, the discussion should help students see the connection between graphical and numerical representations (where is the mean located on the dot plot?); notice that the spread is not visible in the table (Messi's 73 goals in 2011/12 is almost an "outlier" compared to the goals he scored in the other seasons); graphs make trends over time visible (can easily track how the number of goals changed for each player over the seasons). Teachers can use the discussion to help students become more familiar with accurate statistical vocabulary (using words such as distribution, variability, spread of the distribution, range, mean). If a plot did not surface in the student work, the teacher might ask, Is there another way to plot these data? How can you change from a representation we have to a different representation? What advantages does the new representation offer?

Assessment

In considering the assessment of students as they work on more open-ended statistical tasks, it is important to focus on both the process and the product of the work. Using formative assessment strategies, teachers should consider how students demonstrate their knowledge of the statistical concepts, as well as the refinement of that knowledge as they progress through the task. Teachers should have specific goals in mind related to both content and process. For examples of rubrics for each stage of this project, the reader should refer to the report: http://www2.famaf.unc.edu.ar/institucional/biblioteca/trabajos/6085/17150.pdf.

Closing Statement

The intention of this brief is to provide a resource for teachers who wish to engage their students in rich statistical tasks. Recognizing that the area of statistics might be new to many teachers, the brief also calls for the support of teachers as they look to build their content and their pedagogical knowledge.

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