Day 1: Flip The Script

Welcome to PCMI! We know you'll learn a great deal of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How do others think about this question?
- **Be excellent to each other.** Believe that you have something to learn from everyone else. Remember that everyone works at a different pace. Give everyone equal opportunity to express themselves. Don't be afraid to ask questions.
- **Teach only if you have to.** You may feel the temptation to teach others in your group. Fight it! We don't mean you should ignore your classmates but give everyone the chance to discover. If you think it's a good time to teach your colleagues about eigenvalues, think again: the problems should lead to the appropriate mathematics rather than requiring it.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff. Check out the Opener and the Important Stuff first. All the mathematics that is central to the course can be found and developed there. *That's* why it's Important Stuff. Everything else is just neat or tough. Each problem set is based on what happened the day before.

PCMI teachers have solved two previously unsolved problems presented in these courses.

When you get to Day 3, come back and read this again.

Will you remember? Maybe . . .

Opener

1. **a.** Each of you will flip a coin 42 times. Write down H or T in the order that you got it.

42 times? That's already the answer.

b. The 42 flips can be grouped into 40 three-flip sequences. Say you got HTTHHHT... That would give these three-flip sequences: HTT, TTH, THH, HHH, HHT, etc. Tally up the number of times you got each three-flip sequence.

"You got H T T H H H T"

c. Predict which of the eight three-flip sequences would occur most often in the long run.

d. Combine the tallies at your table. One person should enter the combined data here:

`http://bit.ly/pcmi2016`

Note: not literally "here". There.

Important Stuff

2. Sahar is about to flip a coin 42 times. Predict how many of her 40 three-flip sequences will be HTT.

3. Of families with three children, which is more common: a family with three girls, or a family with two girls and one boy?

4. A family has two children, both boys: Braeden and Brendon. What is the probability that the family's next child will be a third boy?

5. Here's a recursive definition for the sequence $0, 5, 10, 15, 20, \ldots$:

$$t(n) = \begin{cases} 0, & \text{if } n = 0 \\ t(n-1) + 5, & \text{if } n > 0 \end{cases}$$

$t(n) = t(n-1) + 5$ means $t(1)$ equals $t(0)$ plus 5. And $t(2)$ equals $t(1)$ plus 5. And $t(3)$ equals $t(2)$ plus 5. And $t(4)$ equals $t(3)$ plus 5. And . . .

a. What is $t(10)$?

b. For some number chai, $t(\mathsf{chai}) = 95$. Find chai.

Be careful when calculating $t(\mathsf{oolong})$. The answer may be too long.

6. Here's another recursive definition.

$$h(n) = \begin{cases} 100, & \text{if } n = 0 \\ 0.5 \cdot h(n-1), & \text{if } n > 0 \end{cases}$$

Complete this table for $h(n)$ and the running total of all $h(n)$, keeping your answers accurate to two decimal places.

This function distinguishes between the halves and the halve-nots.

n	$h(n)$	Total
0	100	100
1		150
2		
3		
4		
5		
6		
7		
8		
9		
10		

Neat Stuff

7. Nataša's favorite sequence starts with $N(0) = 7$ and $N(1) = 4$. After that, each term is the opposite of the sum of the previous two terms. Write the first ten terms of this sequence.

We heard her call this the "neganacci" sequence.

8. Of families with 4 children, which is most common: all 4 kids the same (all boys or all girls), exactly 3 the same, or 2 of each?

9. Find the probability that when you flip a coin 10 times, the sequence HH *never* appears.

10. Alex, Angelina, Chris, Danilsa, Deva, and Manish go to dinner every night and play "credit card roulette": the waiter picks one of their six credit cards at random to pay for the meal.

Problem 10 didn't make any sense until these six formed a Diners Club.

a. What is the minimum number of meals it will take until each of them has paid at least once, and what is the probability of this occurring?

b. What is the maximum number of meals it will take until each of them has paid at least once? Uh oh.

c. What is the *mean* number of meals it will take until each of them has paid at least once?

11. Here's another recursive definition.

$$D(n) = \begin{cases} 5, & \text{if } n = 0 \\ 2 \cdot D(n-1) - 3, & \text{if } n > 0 \end{cases}$$

a. Determine $D(10)$ and a way to calculate $D(n)$ quickly for any n.

b. What value(s) of $D(0)$ would make $D(10) = D(0)$?

12. Kayleigh offers you these two games:

Game 1: You roll a die four times. If you roll a six any of the four times, you win.

Game 2: You roll a pair of dice 24 times. If you roll boxcars (double sixes) any of the 24 times, you win.

Aside from the fact that Game 2 takes longer to play, which of these games would you rather play to win? Or do both games have the same chance of winning?

Problem 12 is much older than the Diners Club! It was originally solved by Pascal.

Tough Stuff

13. Find the probability that when you flip a coin 42 times, the sequence HHH *never* appears.

14. Find the probability that when you flip a coin 42 times, the sequence HHT *never* appears.

15. What is the mean number of meals it will take until the six friends from Problem 10 each pay at least *twice*? At least n times?

Only members of the Diners Club can order the exclusive sandwich.