Day 3: Flipping Out Loud

Opener

1. It's time to play *Beat Cal*! You'll chase Cal to see whose three-flip sequence comes up first. Cal has selected HHH.

Stanford alumni approve this message.

- **a.** At your table, use each of the other seven three-flip sequences to compete against Cal three times.
- **b.** One person at your table should enter the number of wins against Cal for each sequence here:

  `http://bit.ly/pcmi2016`

- **c.** Based on the data, which sequence appears to be the best choice in *Beat Cal*? Explain why.

Your *table* will play a total of 21 games against Cal. Divvy up the work!

People flip a coin in mysterious ways, maybe just falls off of your hand . . .

Important Stuff

2. Nicole has 4 sheets of paper and wants to share them with Mary, Mary Elizabeth, and Mandy. She starts by handing everyone a full sheet, keeping one.

This problem involves a total of four people.

Then Nicole realizes she doesn't want a full sheet. She breaks her sheet into four equal pieces and shares them, keeping one.

Then Nicole realizes she doesn't want a full $\frac{1}{4}$ sheet. She breaks her $\frac{1}{4}$ sheet into four equal pieces and shares them, keeping one . . .

- **a.** This keeps happening. How much paper will Nicole eventually end up with?
- **b.** How much paper will Mary, Mary Elizabeth, and Mandy each end up with?
- **c.** Write an infinite sum based on all the pieces of paper Mandy ends up with.

Nicole has maybe a 25% case of separation anxiety.

Mandy, you found paper right where you are. Mandy, you found paper right where you are.

3. Here's another recursive definition.

$$V(n) = \begin{cases} 1, & \text{if } n = 0 \\ \frac{1}{5} \cdot V(n-1), & \text{if } n > 0 \end{cases}$$

- **a.** Find each value: $V(0)$, $V(1)$, $V(2)$, $V(3)$.
- **b.** Find the value of

$$V(0) + V(1) + V(2) + V(3) + \cdots + V(10) + \cdots$$

Vince has some paper.

4. Here's another recursive definition, but this time it's for three functions at once!

$$a(n) = 0.5b(n-1) + 0.5c(n-1) \qquad \text{with } a(0) = 0.6$$
$$b(n) = 0.5a(n-1) + 0.5c(n-1) \qquad \text{with } b(0) = 0.3$$
$$c(n) = 0.5a(n-1) + 0.5b(n-1) \qquad \text{with } c(0) = 0.1$$

a. Complete each of these tables.

Is it the grand finale now? *No!*

$a(1)$	
$b(1)$	
$c(1)$	

$a(2)$	
$b(2)$	
$c(2)$	

$a(3)$	
$b(3)$	
$c(3)$	

b. Predict the values in this table:

After solving this, clap your hands, but don't give up 4 goals before halftime.

$a(10)$	
$b(10)$	
$c(10)$	

5. The sequence $h(0), h(1), h(2), \ldots$ is defined recursively by

For example, $h(2)$ is $h(1)$ plus 12 times $h(0)$. Then . . .

$$h(n) = h(n-1) + 12h(n-2), \qquad \text{if } n > 1.$$

Six different starting pairs for $h(0)$ and $h(1)$ have been chosen.

$h(0)$	1	0	1	10	11	10
$h(1)$	0	1	1	0	1	40
$h(2)$	12					
$h(3)$	12					
$h(4)$						
$h(5)$						

Complete the table above without any technology. Look for patterns to make your work easier. Share patterns with your tablemates.

Sequences add up in mysterious ways, maybe it's all part of a plan.

6. Revisit $h(n)$, but this time with $h(0) = a$ and $h(1) = b$.

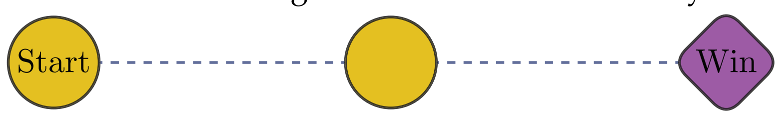
Kiss me under $12a + 13b$ stars . . .

$h(0)$	a
$h(1)$	b
$h(2)$	12a+b
$h(3)$	
$h(4)$	
$h(5)$	

Refer back to Problem 5. What do you notice?

Neat Stuff

7. Let's revisit Cesar's game from Problem 7 on Day 2.



Let $a(n)$, $b(n)$, and $c(n)$ be the probability of Cesar being on the first, second, or third spaces above after the nth coin flip. Cesar starts the game on the first space: $a(0) = 1$, $b(0) = 0$, and $c(0) = 0$.

'Cause honey my tree diagrams are so tall, like evergreens . . .

a. Write recursive definitions for $a(n)$, $b(n)$, and $c(n)$ in terms of $a(n-1)$, $b(n-1)$, and $c(n-1)$.

b. Use these definitions to complete the tables.

$a(1)$	
$b(1)$	
$c(1)$	

$a(2)$	
$b(2)$	
$c(2)$	

$a(8)$	
$b(8)$	
$c(8)$	

c. What is the probability that Cesar *won't* have won after 8 flips?

8. Cal wises up and picks THH instead. Is there a sequence you could pick that will beat THH more than 50% of the time?

9. Revisit $h(n)$, but this time with $h(0)$ and $h(1)$ unknown.

Oh, this must be the grand finale now. *NO!*

a. If $h(2) = 43$ and $h(3) = 127$, find $h(0)$ and $h(1)$.

b. If $h(3) = -1$ and $h(5) = 119$, find $h(0)$ and $h(1)$.

c. Suppose $h(5) = -3 \cdot h(4)$. What can you figure out about the sequence $h(0), h(1), h(2), \ldots$?

10. A sheet of A0 paper is built so it can be cut in half to form two A1 sheets, both scaled copies of the A0 sheet.

a. Give one possible length and width for an A0 sheet, knowing it can be cut in half to form A1.

b. The area of an A0 sheet is 1 square meter. Find its dimensions.

c. If you had one each of A0, A1, A2, A3, etc., how much paper would you have?

And baby, my heart couldn't fit on an A23 . . .

11. In the "Credit Card Roulette" problem from Day 1, how many dinners will it take (on average) for at least two different people to pay? For at least three different people to pay?

12. Write a recursive rule for $c(n)$ that fits the sequence $1, 2, 11, 43, 184, 767 \dots$

13. Marie will flip a coin until she sees heads come up on consecutive tosses. On average, how many flips will it take for this to happen?

14. Blake will roll a die until he sees a 6 come up on consecutive rolls. On average, how many rolls will it take for this to happen?

It's a day late, but happy $P(A|B) = P(A)$ Day!

15. Which is more likely to occur for Cal: HHH appears for the first time on the fourth flip, the fifth flip, or the sixth flip?

HHH prefers to call himself "The Game", and was solid at Wrestlemania 2^5.

16. Shannon will flip a coin until she sees HHH come up. On average, how many flips will it take for this to happen?

Juno what satellite is now in orbit around Jupiter?

Tough Stuff

17. On average, how many coin flips will it take for you to flip n heads in a row?

18. Cal picks HTH and you pick the best possible three-flip sequence to play against him. What is the probability that you will win, and how long on average will the game take?

Wait, was that the finale?