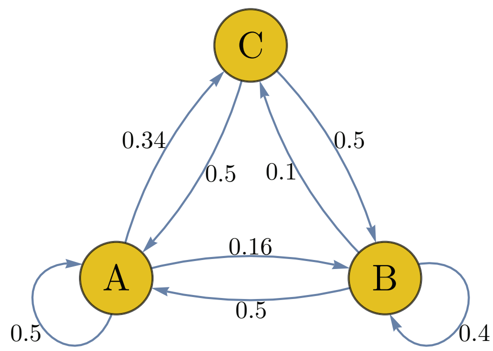
Day 4: Mark of PCMI

Opener

1. Mark found some information on how PCMI used to handle table changes. Back in the day, there were only three tables, and a huge number of people attended PCMI! Here's a visual of the table change rules:

We're not saying the information Mark found is *true*, but he did find it, probably somewhere near a crater.



The arrow from A to B has a 0.16 above it, meaning that 16% of the people at Table A were reassigned to Table B. The arrow from A to A has a 0.5 next to it, meaning that 50% of the people remained at Table A. Here is the same info in tabular form.

Back then, it was possible for a person to be assigned back to the same table. And the table leaders had it tougher.

	from A	from B	from C
to A	.5	.5	.5
to B	.16	.4	.5
to C	.34	.1	0

a. Before the first table change, 60% of participants sat at Table A, 10% at Table B, 30% at Table C. After the first table change, what % of participants were at Table A? Table B? Table C?

b. After a second table change, what percentage of participants were at Table A? Table B? Table C?

c. Fill in the blanks to build the recursive definitions for $A(n)$, $B(n)$, and $C(n)$.

This is clearly before Carol's time at PCMI. She would never let this sort of table disparity slide.

$$A(n) = \boxed{} \cdot A(n-1) + \boxed{.5} \cdot B(n-1) + \boxed{} \cdot C(n-1)$$

$$B(n) = \boxed{.16} \cdot A(n-1) + \boxed{} \cdot B(n-1) + \boxed{.5} \cdot C(n-1)$$

$$C(n) = \boxed{.34} \cdot \qquad\qquad + \boxed{} \cdot \qquad\qquad + \boxed{} \cdot$$

Important Stuff

2. Here are three characters' attributes of strength (STR), intelligence (INT), dexterity (DEX), and hit points (HIT).

Tonight, Pinky, we shall take over the world by spreading Lyme disease and shouting about spoons!

	Pinky	The Brain	The Tick
STR	3	4	20
INT	1	21	1
DEX	5	18	6
HIT	20	7	99

Oyinka wants to create another character, Tiny Pink Breach Kitteh, a precise mix of the three characters: 20% Pinky, 35% The Brain, and 45% The Tick. Calculate the attributes of Oyinka's character.

Oh, you got a better anagram? Go for it.

$$\text{STR} = \boxed{11} = .20 \cdot \boxed{3} + .35 \cdot \boxed{4} + .45 \cdot \boxed{}$$

$$\text{INT} = \boxed{8} = .20 \cdot \boxed{} + .35 \cdot \boxed{} + .45 \cdot \boxed{}$$

$$\text{DEX} = \boxed{} =$$

$$\text{HIT} = \boxed{} =$$

They're not so dinky, they're Pinky and the Brai-ain and the Tick.

3. Follow these steps to calculate this product of a matrix and a vector on an TI-Nspire CX:

A matrix is just an array of numbers. A vector is a matrix that has only one row or column.

$$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} .20 \\ .35 \\ .45 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

a. Press [⌂ on]. If not in calculator mode, press [▦].

b. If someone had previously been in the midst of typing in a calculation to be performed, clear it by pressing [ctrl] then [del].

Please do not push ALT in between. Also, there is no ALT key on an Nspire.

c. Press [templates] to display a menu of templates. Use the directional arrows below the screen to highlight [matrix]. Press [enter] to select it.

d. A screen titled "Create a Matrix" will appear. Press [del] and [4] to change the number of rows to four. Press [tab] twice to highlight the "OK" button then press [enter] to select it.

When creating a matrix, be careful that it does not enslave humanity.

e. Fill in each of the 12 entries of the matrix by typing each one and pressing [tab] to move to the next entry. After the final entry, press [tab] to move your cursor to the right of the matrix. Do not press the multiplication key as it is not needed.

TaB is the official beverage of the Midway Awesome-Totes.

f. Type in the vector by pressing and choosing again. This time, set the number of rows to 3 and the number of columns to 1. Fill in the numbers in that vector as before.

g. Press [enter] to calculate the product of the matrix and vector. Write the answer in the space on the previous page. What do you notice?

I noticed it's the third page already and this is Problem 3.

4. Multiply these. Use the Nspire if you would like to, but look for ways to save time and energy.

a. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} .6 \\ .1 \\ .3 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

b. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} .5 \\ .286 \\ .214 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

c. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$

d. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} =$$

Since it's National Fried Chicken Day, ask Bowen about the time he was in a KFC commercial, and how good chicken tastes on the 20th take. Or ask Darryl why restaurants shouldn't offer him all-you-can-eat fried chicken.

e. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

f. $$\begin{bmatrix} .5 & .5 & .5 \\ .16 & .4 & .5 \\ .34 & .1 & 0 \end{bmatrix} \begin{bmatrix} A(n-1) \\ B(n-1) \\ C(n-1) \end{bmatrix} =$$

5. Let $|x| < 1$ and

$$S = 1 + x + x^2 + x^3 + x^4 + \cdots$$

a. If $x = \frac{1}{2}$, what is the value of S?

b. If $x = \frac{1}{4}$, what is the value of S?

c. Rewrite the right side of the equation above so it looks like this:

$$S = 1 + x \cdot (\qquad\qquad\qquad\qquad\qquad)$$

d. Use a substitution to write this equation for S in terms of S.

e. Show that

$$S = \frac{1}{1-x}$$

The S clearly doesn't stand for "stop changing value".

No, $S = S$ is not what we mean here, yo.

6. Fill in this table with the missing words. "Matrice" is NOT a word!

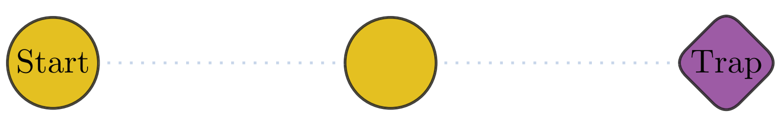
Plural	Singular
matrices	
vertices	
indices	
	helix
	radius
	Prius

It's a word now, it's in the problem set!

Neat Stuff

7. Chris creates his own version of Cesar's game from Problem 7 on Day 2. In Chris's game, he starts in the circle marked "Start" below. He gets to flip a coin as many times as he wants. When he flips a head, he moves forward one space. When he flips a tail, he moves back one space (if possible). If he reaches the "Trap" space, flipping either a head or tails will not change his position.

Cesar is very likely to file a cesar-and-desist letter.



Let $a(n)$, $b(n)$, and $c(n)$ be the probability of Chris being on the first, second, or third spaces above after the nth coin flip. Chris starts the game on the first space: $a(0) = 1$, $b(0) = 0$, and $c(0) = 0$.

a. Add arrows to the diagram above in the style of today's Opener. Label the arrows with the correct transition probabilities.

ARROW'D!!!

b. Write recursive definitions for $a(n)$, $b(n)$, and $c(n)$ in terms of $a(n-1)$, $b(n-1)$, and $c(n-1)$.

c. Rewrite your recursive definitions using matrices.

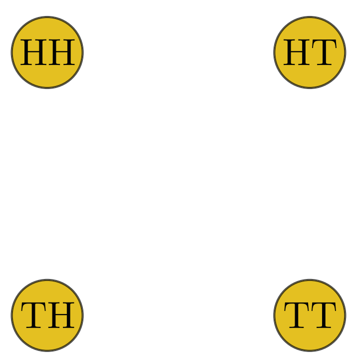
d. Calculate each vector.

$$\begin{bmatrix} a(1) \\ b(1) \\ c(1) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \qquad \begin{bmatrix} a(2) \\ b(2) \\ c(2) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \qquad \begin{bmatrix} a(8) \\ b(8) \\ c(8) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

e. What is the probability that Chris *won't* be in the trap after 8 flips?

8. This diagram gives the four possible two-flip sequences of heads and tails. Add arrows to the diagram in the style of today's Opener. Label the arrows with the correct transition probabilities.

You just flipped HT! What are you going to do next?



HAMMER!! I don't know what that was, but it was funny.

9. Draw a diagram for transitioning between three-flip sequences.

10. How can the diagram in Problem 9 help you beat Cal (over the long term), no matter what he chooses?

I don't know why this is all the way back here. Clearly, beating Cal is important stuff.

11. Gabe claims to have a foolproof way to win at rock-paper-scissors: he randomly picks rock 50% of the time, paper 25% of the time, and scissors 25% of the time.

a. Show that this isn't foolproof by finding a strategy that will beat Gabe more often than it loses.

b. Now find a strategy that is better than *your new strategy*.

c. How well does Gabe do against the strategy that beat your new strategy?

Beat Gabe is not the chant of any colleges, thankfully.

12. Draw a diagram for attempting to flip three heads in a row, then use this diagram and some other tools to find the probability of flipping HHH within 10 flips.

13. Draw a diagram for attempting to flip the sequence TTTH, then use this diagram and some other tools to find the probability of flipping TTTH within 20 flips.

This is better known as the Talk To The Hand sequence.

14. Team Set Theory is 60% likely to win a game against the Awesome-Totes. The two teams play a best-of-seven series. How likely is it that Team Set Theory wins the series?

This probability might be 1, because an asymptote frequently tends toward zero . . . or perhaps even negative infinity . . .

Tough Stuff

15. Find the probability of completing the game *Hi Ho! Cherry-O* within the first four turns.

16. Cal picks THH and you pick the best possible three-flip sequence to play against him. What is the probability that you will win, and how long on average will the game take?

17. **a.** Consider a graph of four nodes connected as a square. Let a, b, c, d be whole numbers at each node (going around the square in order). Suppose that the number at each node divides the sum of its nearest neighbors. In other words, b is a factor of $c + d$. And, also assume that a, b, c, d share no common factor other than 1. How many different solutions (up to rotational symmetry) exist?

b. Extend to a cube.

c. Extend to a n-dimensional hypercube.