Day 7: Wish for Fish

Opener

1. It's time to play ... *Wheel of Fish*! Let's bring up our contestants, Becky and Cindy! They'll each spin the fabulous Wheel of Fish. Because she won the coin toss, Becky will get to spin the wheel three times and Cindy will get to spin the wheel twice. Cindy spins first.

This is the popular followup to the quickly-canceled *Wheel of Urchin*.

a. What is the probability that Cindy *will not* land on the "10 fish" space on any of her two spins?

b. What is the probability that Becky *will* land on the "10 fish" space on any of her three spins?

Ahhh, red Swedish fish. Mmmmm, very tasty. Okay, Becky, listen carefully. You can hold on to your red fish . . . or you can go for what's in the box that Darryl is bringing down the aisle right now! What's it gonna be?

Important Stuff

2. Kate spins the Wheel of Fish hoping for 10s. What is the expected number of times Kate will land on a 10 in . . .

a. . . . 1000 spins?

b. . . . 100 spins?

c. . . . 10 spins?

d. . . . 1 spin?

3. How could you use coins to act like a spin of the Wheel of Fish? Be super specific.

4. Irene will spin the Wheel of Fish repeatedly until she lands on the "10" space, hitting the fish jackpot.

The fish jackpot? Seriously, that's a thing? Google says it's got nearly 7,000 hits, so yeah, I guess it is.

By the way, you already did this Snap Judgment.

a. **Snap Judgment!** On average, how many times should she expect to spin the wheel until she lands on the 10 space? You have 15 seconds to answer! Guess! Go with your gut!

b. Use `http://bit.ly/wheel-of-fish` to spin the wheel repeatedly. Count the number of spins it takes until you land on the 10. Write that here: ____________

c. Play this again! Write the number of spins here: ____________

d. Average your data with your tablemates. One person should enter the averaged data here:

`http://bit.ly/pcmi2016`

I know, the Wheel of Fish is totally mesmerizing. But it's time to move on.

5. Renee will flip a coin repeatedly until she flips two heads in a row.

a. **Snap Judgment!** On average, how many times should she expect to flip the coin until she flips HH? You have 15 seconds to answer! Guess! Go with your gut!

Don't make us put another spelling problem in Important Stuff! There's no E in "judgment". Oh, crap, there is.

b. You play this game too. Record the number of flips until you get HH. Write the number of flips here: Write that here: ____________

c. Play this again! Write the number of flips here: ____________

For example, if you flipped HTHTHH, record 6. If you flipped TTTTTTTTTTTTTTTTHH, seek medical help immediately. Ask your doctor if coin flipping is right for you.

d. Average your data with your tablemates. One person should enter the averaged data here:

`http://bit.ly/pcmi2016`

6. Multiply these. Use the Nspire if you would like to, but look for ways to be lazy.

These numbers might seem familiar . . . Also, this is a good time to let you know you that on the Nspire, you can store a matrix or vector as a variable!

a. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **g.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **h.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **i.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

Little known Slurpee fact: in Oklahoma City, 7-Eleven stores sell "Icy Drink", which looks and tastes just like a Slurpee but is not called a Slurpee. These 7-Eleven stores are run by a completely different company!

d. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **j.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

e. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **k.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

f. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ **l.** $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}^2\begin{bmatrix} 2 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

7. Multiply these. Again, look for ways to be lazy.

Keep going, Amy!

a. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 0 & 10 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \end{bmatrix}$

c. $\begin{bmatrix} .5 & .5 & 0 \\ .5 & 0 & 0 \\ 0 & .5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

d. $\begin{bmatrix} .5 & .5 & 0 \\ .5 & 0 & 0 \\ 0 & .5 & 1 \end{bmatrix} \begin{bmatrix} .5 & .5 \\ .5 & .25 \\ 0 & .25 \end{bmatrix} = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$

Little known Slurpee fact: there are more than twice as many 7-Eleven stores in Japan than there are in the United States: over *18,000* stores. And yes, they sell Slurpees, what kind of question is that?

8. Calculate the value of

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

Neat Stuff

9. Fill in the missing blanks. Explain what was done to go from one line to the next.

$$S = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{16} + \cdots$$

$$2S = 1 + \left(2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{8} + \cdots\right)$$

$$2S = 1 + \underbrace{\left(1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + \cdots\right)} + \underbrace{\left(\frac{1}{2} + \boxed{} + \boxed{} + \cdots\right)}$$

$$2S = 1 + \boxed{} + \boxed{}$$

Use the work above to calculate the value of S.

10. If Becky and Cindy had an infinite number of tries at spinning the Wheel of Fish to land on a 10, then their game would have looked like this:

Spin, then win! You cannot win if you do not spin! Also, you can't have any pudding if you don't eat your meat.

a. Write transition probabilities next to the arrows in the diagram above.

b. Create a transition matrix T for this situation.

c. Multiply

$$T^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $n = 1, 5, 10, 20$. What will the result approach as n keeps increasing?

d. Interpret these results based on this context.

Little known Slurpee fact: Slurpees are different in Canada, made without compressed air. They are even more yummy. One of the course instructors may have driven to Canada exclusively to buy a Slurpee.

11. Make a game that is about one-third likely to be won. Explain clearly how the game is played, and what the winning condition is. The best games are simple to play but complex in their potential outcomes. Don't worry too much about making the winning probability exactly one-third.

One game would be "Roll a die and if it comes up 1 or 2, you win." But there are more interesting games!

12. Make a game using one or more six-sided dice with exactly a one-*fifth* probability of success.

13. Kate and Renee are chasing Pokémon all over Midway. There are 10 types of local Pokémon, and any encounter is a random choice of one of the 10.

Sadly, the accordion-playing bear is not a Pokémon. However, it *is* a Poképspot. Whatever *that* is.

a. What is the probability that their second Pokémon encounter is with a *different* Pokémon than the first?

b. What is the probability that after acquiring two different Pokémon, their next encounter is with a *different* Pokémon than the first two?

c. Determine the expected number of encounters Kate and Renee will need to have until they have collected three different Pokémon.

d. On average, how many encounters will Kate and Renee need to . . . what's that phrase . . . acquire every one of them?

14. Use similar tactics to Problem 9 to find this infinite sum:

$$1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{16} + 3 \cdot \frac{9}{64} + 4 \cdot \frac{27}{256} + \cdots$$

15. On *Super Wheel of Fish* the wheel has ten spaces: the numbers 1 through 9 and "Game Over".

The *Wheel of Fish* tournament of champions is always compelling! And smelly.

a. What is the average number of fish a player can expect to win?

b. The producers are considering changing "Game Over" to "Go Home Empty". If this happens, what strategy should players use to decide when to stop?

There's even a Slurpee documentary! It's called The Frozen City, about the place that drinks the most Slurpees per person: Winnipeg, Manitscolda.

16. On a table are three coins, all heads-up. Justin picks a coin at random and turns it over. This process repeats.

a. After three turns, what is the average number of coins that are heads?

b. What happens in the long run?

Tough Stuff

17. Just like the previous problem, there are three coins, all heads-up. Katherine rolls a die. If it's a 1, she turns over the first coin. If it's a 2 or a 3, she turns over the second coin. Otherwise, she turns over the third coin.

a. After three turns, what is the average number of coins that are heads?

b. What happens in the long run?

You blew it, Amy! Now we'll all be stuck teaching math in the Wild West of the 19th Century. On the plus side, we can introduce Common Core early, as kids learn the 3 R's: ropin', ridin', and 'rithmetic. Sorry, this probably made sense to almost nobody ... which is pretty typical of most sidenotes, actually.

18. Find a finite-length game based entirely on coin flipping that has probability of success $p = \frac{1}{3}$, or show that such a game is impossible.

19. A bag contains n red chips and n black chips. You repeatedly draw a chip at random from the bag. Black chips are replaced, but red chips are removed from the bag forever. On average, how many times will you have to draw from the bag to remove all the red chips?

20. OK, the reality is that Kate and Renee are chasing 150 different Pokémon types. If every type is equally likely to be encountered, how many encounters (on average) will it take them to . . . what's that phrase . . . obtain a complete set?

I'll bet they haven't even found a Lickitung yet!

21. Figure this out.

$$\tan\left(\frac{3\pi}{11}\right) + 4\sin\left(\frac{2\pi}{11}\right) = \sqrt{11}$$