Day 8: Fish and Chips

Opener

1. It's time once again to play ... *Wheel of Fish*! Let's bring up our contestants, Elissa and Jeff! Jeff won the coin toss, so he'll spin twice, while Elissa will spin once.

That last episode busted our budget.

a. Complete this expression for the *expected value* of the number of fish given away to Elissa:

$$\text{EV} = 1 \cdot \boxed{} + 2 \cdot \boxed{} + 3 \cdot \boxed{} + 10 \cdot \boxed{}$$

b. What are the different amounts of fish Jeff can win, and how likely is each possibility?

c. What is the expected value of the number of fish given away to Jeff?

d. What combination of two spins could allow Jeff to obtain the number of fish from part c?

Wheel of Fish also clearly beats *Wheel of Phish* in the ratings. An episode of *Wheel of Phish* is a jam session that can last hours, but might also try to steal your identity.

Important Stuff

2. A typical episode of *Wheel of Fish* has 20 spins.

a. How many times, on average, will players hit the 10-fish jackpot?

b. How many fish, on average, will the show give away?

3. Kiaundra will flip three coins and count the number of heads that appear.

I suppose each person has their preference between fish heads and fish tails.

a. What are the different numbers of heads Kiaundra can flip, and how likely is each possibility?

b. What is the average number of heads Kiaundra should expect to flip?

c. What combination of flips could allow Kiaundra to obtain the number of flips from part b?

4. Chance will flip one coin and count the number of heads that appear. What is the average number of heads Chance should expect to flip?

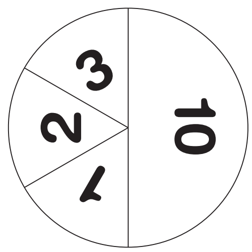
Man, what are the chances of that? I'll bet as a math teacher Chance would never have heard such a joke before. Never.

5. Calculate the value of

$$\frac{1}{4}+\frac{3}{4^2}+\frac{3^2}{4^3}+\frac{3^3}{4^4}+\cdots$$

6. Matthew draws the Wheel of Chips:

You know, you can't eat fish without chips. Or fries, or whatever "chips" are.



For those of you are really picky: the sectors for 1, 2, 3 have the same area, and the sector for 10 is a half-circle. Also, eat your vegetables.

What is the expected number of chips earned from one spin of this wheel? Two spins?

7. Hannah will spin the Wheel of Fish until she lands on the "10" space, then stops.

NOT the Wheel of Chips, that's just some weird thing Matthew made up.

a. What is the probability that Hannah gets it on the first try?

b. . . . on the second try? (What has to happen?)

c. . . . on the third try?

d. . . . on the fourth try?

8. Complete this expression for the expected number of spins of the Wheel of Fish until Hannah lands on the 10. Oh noes, the answer is an infinite series:

What's in the boxes?! Hopefully not the same thing Becky won last time.

$$EV = 1\cdot\boxed{\quad\quad}+2\cdot\boxed{\quad\quad}+3\cdot\boxed{\quad\quad}+4\cdot\boxed{\quad\quad}+\cdots$$

9. Fill in the missing blanks. Explain what was done to go from one line to the next.

$$S = 1\cdot\frac{1}{4}+2\cdot\frac{1}{4}\cdot\frac{3}{4}+3\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2+4\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^3+\cdots$$

$$\frac{4}{3}S = \frac{1}{3}+\left[2\cdot\frac{1}{4}+3\cdot\frac{1}{4}\cdot\frac{3}{4}+4\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2+\cdots\right]$$

$$\frac{4}{3}S = \frac{1}{3}+\underbrace{\left[1\cdot\frac{1}{4}+2\cdot\frac{1}{4}\cdot\frac{3}{4}+3\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2+\cdots\right]}+\underbrace{\left[\frac{1}{4}+\boxed{\quad}+\boxed{\quad}+\cdots\right]}$$

$$\frac{4}{3}S = \frac{1}{3}+\quad\boxed{\quad}\quad+\quad\boxed{\quad}$$

Use the work above to calculate the value of S.

Neat Stuff

10. Draw a Wheel of Peas with the same numbers (1, 2, 3, 10) so that the expected number of peas that will be earned in one spin is 3.

Yep, this is the proper vegetable to go with fish and chips . . .

11. Expand this:

$$\left(\frac{1}{4}x^1 + \frac{1}{4}x^2 + \frac{1}{4}x^3 + \frac{1}{4}x^{10}\right)^2.$$

How does this calculation help you answer Problem 1?

We don't mean this.

12. Build a polynomial that models one spin of the Wheel of Chips, and use it to determine the probability that on two spins of the Wheel of Chips, at least 5 chips are earned.

We believe pigeons prefer chips over peas, but only if at least one pigeon gets to eat two chips. Chris would know more about this.

13. Build a polynomial that models one spin of the Wheel of Peas, and use it to determine the probability that on two spins of the Wheel of Peas, at least 5 peas are earned.

14. Jen rigs a pair of dice so that each 6 is rolled with probability 0.5 and the other five faces are each rolled with probability 0.1.

a. Write a polynomial you could use to model rolling this unfair six-sided die.

b. What's the most likely total rolled on four dice, and how likely is it?

15. Generalize Problems 5 and 9. Interpret your result in the context of repeated attempts at playing a game.

16. The grand finale on *Wheel of Fish* is the Gauntlet. To win, Ann must roll a six, then flip heads, then spin a 10 on the Wheel of Fish. Each task is repeated as many times as necessary to get the job done.

a. What is the probability that Ann will complete the Gauntlet in three turns?

b. What is the probability that Ann will complete the Gauntlet within ten turns?

c. What is the average number of turns it will take a player to complete the Gauntlet?

Before the Gauntlet begins, the player must choose whether they want to be a Warrior, Valkyrie, Elf, or Wizard. Rolls, flips, and spins are "fair" and happen in order, so the Gauntlet might look like this: roll, roll, roll, roll, flip, flip, flip, spin, spin.

17. Use the tactic from Problem 16 to help you find the average number of Pokémon Renee must encounter until each of them pays for dinner at least once. Sorry, we meant how many until grabbing at least one of each of the 10 types. Whatever.

With all the money that game is making, the Pokémon should easily be able to afford buying dinner.

18. Expand this:

$$\left(1+r+r^2+r^3+\cdots\right)\left(1+r+r^2+r^3+\cdots\right) = 1+\boxed{}\cdot r+\boxed{}\cdot r^2+\boxed{}\cdot r^3+\cdots$$

19. Problem 9 is just one way to calculate the value of that sum. Find some other ways.

Sum other ways?

20. Find the probability that players win exactly 80 fish in 20 spins of the Wheel of Fish.

Tough Stuff

21. At 7-Eleven, Anne walks in and buys four items. Without tax, the cost of the four items is \$7.11. Amazingly, the *product* of the four items' costs is also 7.11. What are the costs of the four items?

With correct units, the product of the items' costs is \$\$\$\$7.11, or 7.11 $\$^4$. Sadly, this problem is a day late and a penny short.

22. The Game of Life has a spinner with the numbers 1 through 10 on it. The shortest path from start to finish is 123 spaces, with mandatory stops after space 15 and space 26. On average, how many spins will it take for a player to complete The Game of Life using this path?

Sadly, this means you didn't go to college. But, on the plus side, no promissory notes.

23. This is the sum of all unit fractions with prime denominators:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{11}+\cdots$$

Does this sum converge? If so, to what? If not, can you prove it?

That's from the new Lil Jon song, "If So To What".

24. Figure this out.

$$\tan\left(\frac{\pi}{7}\right)+2\sin\left(\frac{3\pi}{7}\right)\sec\left(\frac{\pi}{7}\right)=\sqrt{7}$$